

Nonthermal Radiation from Type Ia Supernova Remnants

Paul P. Edmon,^{1*} Hyesung Kang,^{2†} T. W. Jones,^{1‡} Renyi Ma,^{3§}

¹*Department of Astronomy, University of Minnesota, Minneapolis, MN 55455, USA*

²*Department of Earth Sciences, Pusan National University, Pusan 609-735, Korea*

³*Department of Astronomy and Space Sciences, Chungnam National University, 305-764, Daejeon, Korea*

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ABSTRACT

We present calculations of expected continuum emissions from Sedov-Taylor phase Type Ia supernova remnants (SNRs), using the energy spectra of cosmic ray (CR) electrons and protons from nonlinear diffusive shock acceleration (DSA) simulations. A new, general-purpose radiative process code, *Cosmicp*, was employed to calculate the radiation expected from CR electrons and protons and their secondary products. These radio, X-ray and gamma-ray emissions are generally consistent with current observations of Type Ia SNRs. The emissions from electrons in these models dominate the radio through X-ray bands. Decays of π^0 s from $p-p$ collisions mostly dominate the gamma-ray range, although for a hot, low density ISM case ($n_{ISM} = 0.003 \text{ cm}^{-3}$), the pion decay contribution is reduced sufficiently to reveal the inverse Compton contribution to TeV gamma-rays. In addition, we present simple scalings for the contributing emission processes to allow a crude exploration of model parameter space, enabling these results to be used more broadly. We also discuss the radial surface brightness profiles expected for these model SNRs in the X-ray and gamma-ray bands.

Key words: radiation mechanisms: non-thermal, acceleration of particles, (ISM:) supernova remnants, (ISM:) cosmic rays

* E-mail: edmonpp@msi.umn.edu

† E-mail: kang@uju.es.pusan.ac.kr

‡ E-mail: twj@astro.umn.edu

§ E-mail: mry@canopus.cnu.ac.kr

1 INTRODUCTION

Diffusive Shock Acceleration (DSA) at astrophysical shocks has become the standard model for production of cosmic rays (CRs) (*e.g.*, Malkov & Drury 2001, and references therein). The CRs observed at the knee ($\sim 10^{15}$ eV) and below are most commonly explained through DSA in the blast waves of Galactic supernova remnants (SNRs) (Blandford & Ostriker 1978; Lagage & Cesarsky 1983; Blandford & Eichler 1987; Drury et al. 2001). Several indirect arguments, including energetics and composition, support this explanation (*e.g.*, Gaisser 2005; Hillas 2006; Ellison et al. 2007).

Direct evidence that at least some SNRs are capable of accelerating electrons and possibly hadrons to at least tens of TeV comes from observations of nonthermal X-ray filaments in some historical SNRs (*e.g.*, Bamba et al. 2006; Parizot et al. 2006) and from TeV γ -rays observed in several shell SNRs (*e.g.*, Funk et al. 2007; Lemoine-Goumard 2007). The nonthermal X-rays are likely to be synchrotron emission from TeV electrons (*e.g.*, Uchiyama et al. 2007). The TeV γ -ray origins, although they obviously require TeV or higher energy charged particles, have less certain origins and are sensitive to several model parameters. Inverse Compton scattering, secondary π^0 decays and bremsstrahlung may all contribute, but at levels that depend on details of the accelerated CR spatial and energy distributions as well as ambient photon, magnetic field and plasma properties. However, recent observations by *Fermi* of several Type II SNRs indicate that secondary π^0 decay from proton-proton interactions is the most likely explanation for the γ -ray emission there, which would observationally confirm the SNR origins of CR protons (Abdo et al. 2010).

In order to test the SNR origins of Galactic CRs, generally, and the accuracy and physical assumptions of DSA model calculations, specifically, it is important to determine from simulations not only the CR distributions, but also the related emissions, so they can be compared with current and anticipated observations (*e.g.*, Blandford & Eichler 1987; Hillas 2005). Indeed several model SNR comparisons of this type have been published (*e.g.*, Ellison & Cassam-Chenaï 2005; Berezhko, Ksenofontov & Völk 2009; Morlino, Amato & Blasi 2009; Zirakashvili & Aharonian 2010). Those demonstrate that observed emissions from some individual SNRs can be explained by the simulations with reasonable model choices, although they do not yet provide unambiguous model tests nor clear confirmation of the specific origins of TeV emissions.

At this early stage of our understanding it is especially useful to explore generally the

nonthermal emissions produced by CRs in common classes of SNR models and how they depend on model parameters. In this spirit we present here an examination of the nonthermal radio to γ -ray emissions for Sedov-Taylor phase Type Ia SNRs, using a set of time dependent simulations that followed the evolution of spherical blasts, including nonlinear DSA at quasi-parallel shocks with thermal leakage injection of CR protons and electrons out of the bulk plasma. The pressure of CR protons is included in the gasdynamic equations, while CR electrons are treated as a test-particle component. The principal differences expected in the electron and proton CR populations come from differences in injection rates from the thermal population and from significant synchrotron/Compton radiative losses experienced by the highest energy electrons, but not the protons. We consider four different SNR models, including a range of uniform external environments and blast energies.

The structure of our paper is as follows. Section 2 provides a brief outline of the simulations that include both CR protons and electrons. A description of the code used to calculate nonthermal emissions as well as results from our emission calculations are presented in §3, while §4 summarizes our conclusions.

2 NONLINEAR DSA SIMULATIONS OF SN IA SEDOV-TAYLOR REMNANTS

2.1 The Spherical CRASH code

In the simulations discussed here the evolution of the CR modified SNR shock is followed using a one-dimensional, spherically symmetric version of the Cosmic RAY SHock (CRASH) gasdynamic-CR code. The basic gasdynamic equations modified to implement nonlinear DSA and full code details can be found in Kang & Jones (2006). CRASH applies adaptive mesh refinement (AMR) techniques and subgrid shock tracking to obtain high spatial resolution close to the blast shock, where it is crucial for converged solution of the coupled gas dynamics and CR transport equations. The spherical CRASH code also incorporates comoving coordinates expanding with the SNR blast. The momentum-dependent CR distribution is evolved through a time-dependent solution of the diffusion convection equation (DCE). CRASH incorporates dynamical backreaction from CRs onto the gas dynamics, including influence of the CR pressure, P_c , and energy exchanges coming from injection of low energy CRs at the gas subshock and also dissipation of Alfvén waves stimulated by streaming CRs. We note that Berezhko and collaborators have extensively studied the similar problem,

implementing a different, unconventional numerical method for the gasdynamics that normalizes the spatial variable by diffusion length at each momentum value and solves the CR kinetic equation iteratively to match the downstream and upstream solutions at the subshock (*e.g.*, Berezhko et al. 1994; Berezhko & Völk 1997; Berezhko, Ksenofontov & Völk 2002). We find that the results of our simulations using the more conventional CRASH code are quite consistent with these previous studies.

Since DSA operates on relativistic electrons of a given p in exactly the same way as it does on protons (i.e., it is exclusively rigidity dependent), the pitch-angle-averaged phase space distribution functions for CR proton and electron components, $f_p(p, r, t)$ and $f_e(p, r, t)$ both obey the DCE,

$$\frac{\partial g}{\partial t} + (u + u_w) \frac{\partial g}{\partial r} = \frac{1}{3r^2} \frac{\partial}{\partial r} [r^2(u + u_w)] \left(\frac{\partial g}{\partial y} - 4g \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \kappa(r, y) \frac{\partial g}{\partial r} \right] + p \frac{\partial}{\partial y} \left(\frac{b}{p^2} g \right). \quad (1)$$

Here g represents either $g_p = p^4 f_p$ or $g_e = p^4 f_e$, $y = \ln(p)$ and $\kappa(r, y)$ is the spatial diffusion coefficient in the radial direction (Skilling 1975). Henceforth, particle momenta, p , of both protons and electrons are expressed in units of $m_p c$, with m_p the proton mass.

The cooling term is $b(p) = -dp/dt = 0$ for protons and

$$\begin{aligned} b(p) &= -\frac{m_p}{m_e} \frac{\sigma_{TC}}{6\pi} \frac{B_{\text{eff}}^2}{m_e c^2} p^2 \\ &= -2.37 \times 10^{-14} \text{s}^{-1} \left(\frac{B_{\text{eff}}}{100 \mu\text{G}} \right)^2 p^2 \end{aligned} \quad (2)$$

for electrons combining synchrotron and IC cooling, where σ_T is the Thomson cross section, and $B_{\text{eff}}^2 = B(r)^2 + B_r^2$ is the effective magnetic field strength including the energy density of the ambient radiation field. We discuss the magnetic and radiation energy densities in §3.2. The code does not include direct backreaction from large scale magnetic fields, nor the pondermotive force of the wave turbulence.

The velocity u_w represents the effective radial motion of scattering centers with respect to the bulk flow velocity, u . Assuming that waves upstream of the gas subshock are predominantly resonantly generated through CR streaming and that the shock has a parallel magnetic field geometry, upstream of the gas subshock u_w is set to the Alfvén speed, $v_A > 0$. The current version of CRASH does not follow the self-consistent evolution of the magnetic field strength through wave-particle interactions. So here we simply set $v_A = B_0 / \sqrt{4\pi\rho(r)}$, where B_0 is the upstream mean magnetic field strength. Downstream, the simulations assume $u_w = 0$, since the Alfvénic turbulence in that region is probably relatively balanced. The transition in scattering center motion, $\Delta u_w > 0$, reduces the effective velocity difference

experienced by CRs across the shock compared to the bulk flow, $\Delta u < 0$, thus reducing the DSA efficiency. Gas heating due to Alfvén wave dissipation is represented by the term

$$W(r, t) = -\omega_H v_A \frac{\partial P_c}{\partial r}, \quad (3)$$

where ω_H is a commonly used dimensionless parameter that controls the degree of dissipation. This dissipation term derives from a simple model in which Alfvén waves are resonantly amplified by streaming CRs in balance with local wave dissipation processes (*e.g.*, Jones 1993). As previously shown in SNR simulations (*e.g.*, Berezhko & Völk 1997; Kang & Jones 2006; Caprioli et al. 2011; Ptuskin et al. 2010), and in more general contexts in earlier work (*e.g.*, McKenzie & Völk 1982; Markiewicz, Drury, & Völk 1992; Jones 1993), Alfvénic drift and precursor heating by wave dissipation both reduce DSA efficiency and associated shock modification. The significance of Alfvén wave dissipation and drift can generally be characterized in terms of the Alfvénic Mach number, *i.e.*, $v_A = u_0/\mathcal{M}_A$, where u_0 is the upstream plasma flow speed relative to the shock. In these simulations $\mathcal{M}_A \gg 1$, so the influence is modest.

Bohm-like spatial CR diffusion in the radial direction was used for both protons and electrons in the simulations; namely,

$$\kappa(r, p) = \kappa_n \cdot p \cdot \left(\frac{\rho_0}{\rho(r)} \right), \quad (4)$$

where $\kappa_n = m_p c^3 / (3e B_0) = 3.13 \times 10^{22} \text{ cm}^2 \text{ s}^{-1} \text{ B}_\mu^{-1}$, B_μ is the upstream magnetic field strength in micro-Gauss, and ρ_0 is the upstream density. The density dependence in this diffusion model represents enhancement of resonant Alfvén wave amplitudes through compression.

Low energy CR protons and electrons were injected at the shock in the same manner through thermal leakage (Gieseler, Jones, & Kang 2000; Kang, Jones, & Gieseler 2002). In this model, thermal ions in a Maxwellian distribution immediately downstream of the gas shock have a finite probability to escape upstream into the low energy CR population, provided they have sufficient upstream-directed momentum to overcome transverse MHD waves generated through the cyclotron instability in the shock transition (Malkov & Völk 1998). This behavior is modeled numerically through a “transparency function”. The function contains one adjustable parameter, $\epsilon_B = B_0/B_\perp$, which compares the upstream magnetic field, B_0 , with the amplitude of postshock, amplified MHD waves that interact with low energy particles, B_\perp . The value $\epsilon_B = B_0/B_\perp = 0.25$ was used for the models considered here. The resulting fraction of thermal protons injected into the CR pool, $\xi \sim 10^{-3}$. It was shown pre-

viously that the DSA efficiency saturates at such a high injection rate (if $\xi > 10^{-4}$ for strong shocks), so except for details near start up, the SNR simulation results are insensitive to ϵ_B . The temperature used for the thermal population was computed from the gas pressure, P_g , and density, ρ , as $T = \mu m_p P_g / (k_B \rho)$ with $\mu = 0.61$, so assuming equal proton and electron temperatures.

Because postshock thermal electrons have smaller gyroradii, compared to thermal protons, the injection rate of electrons should be much smaller. The ratio of CR electron to proton number injected at the shock, $K_{e/p} \sim 0.01$, is commonly assumed, since about 1% of the observed Galactic CR flux near a GeV is due to electrons (Schlickeiser 2002; Reynolds 2008). On the other hand, Morlino, Amato & Blasi (2009) point out that the CR electron flux observed here at Earth is actually a convolution of CR electrons from SNR and other electron accelerators of all ages. It is also entirely possible that CR electron injection, which is not well understood, and acceleration efficiency change dramatically for SNRs of different ages. On the other hand, it is commonly found in models of young SNRs that the best fit to the observational data requires $K_{e/p} \sim 10^{-4}$ (*e.g.*, Berezhko, Ksenofontov & Völk 2009; Morlino, Amato & Blasi 2009; Zirakashvili & Aharonian 2010), so we set $K_{e/p} = 10^{-4}$ in our calculations. However, since the electron population is passive and the SNRs are optically thin in emission bands of relevance here, our resulting emissions can be generally linearly scaled to another preferred value of $K_{e/p}$.

In flows where CR backreaction is important, the formation of a CR precursor compresses and heats the inflowing plasma. These developments can lead to substantial changes in the strength of the dissipative, gas subshock and to the postshock conditions relative to those in a pure gas dynamic shock of the same Mach number. Given these flow modifications in front of the subshock, it is helpful in the following discussion to identify specifically the unmodified, upstream conditions by the subscript '0', the conditions immediately upstream of the gas subshock by the subscript '1', and the conditions immediately downstream of the full shock structure by the subscript '2'.

2.2 Remnant model parameters

Table 1 lists the dynamical parameters for the SNR models: the uniform ambient density, n_{ISM} , the ISM temperature, T_{ISM} , the upstream magnetic field strength, B_0 , and the SN explosion energy E_o . The parameters for S1- S3 (S4) represent warm (hot) phase ISM en-

Table 1. SNR model parameters

Model	n_{ISM} (cm^{-3})	T_{ISM} (K)	E_o (10^{51} ergs)	B_0 (μG)	r_o (pc)	t_o (years)	u_o (10^4 km/s)
S1	0.3	3×10^4	1	30	3.19	255.	1.22
S2	0.3	3×10^4	4	30	3.19	127.	2.45
S3	0.3	3×10^4	1	5	3.19	255.	1.22
S4	0.003	10^6	1	5	14.8	1182.	1.22

Note: The model ISM Alfvén speed, $v_A = 101$ km/s for S1,S2; $v_A = 16.8$ km/s for S3; $v_A = 168$ km/s for S4.

vironments. Models S1, S3, and S4 assume an explosion energy, $E_o = 10^{51}$ erg, while model S2 begins with a blast containing four times this energy, so $E_o = 4 \times 10^{51}$ erg. All models assume there are no pre-existing CRs in the ambient ISM.

The SN ejecta mass is set to $M_{ej} = 1.4M_\odot$. The simulations are intended to follow evolution during the adiabatic, Sedov-Taylor (ST) evolution stage. Thus, convenient normalization variables are $\rho_o = 2.34 \times 10^{-24} n_{ISM} \text{ g cm}^{-3}$ for density, along with ST similarity scales $r_o = [3M_{ej}/(4\pi\rho_o)]^{1/3}$ for length, and $t_o = (\rho_o r_o^5/E_o)^{1/2} = [(3M_{ej}/(4\pi))]^{5/6} \rho_o^{-1/3} E_o^{-1/2}$ for time. The velocity and pressure scales are defined as $u_o = r_o/t_o = \sqrt{4\pi E_o/(3M_{ej})}$ and $P_o = \rho_o u_o^2 = E_o/r_o^3$, respectively. *In order to avoid confusion later, we note here a distinction between ST normalization subscripts, 'o' and initial or upstream condition subscripts '0'.*

It is worth noting that the mass swept up by the blast at time t and radius $r_s = \xi_s r_o (t/t_o)^{2/5}$ is given in the ST solution as

$$M_{sw}(t) = M_{ej} \xi_s^3 (r_s/r_o)^3 = M_{ej} \xi_s^3 (t/t_o)^{6/5}, \quad (5)$$

where $\xi_s \approx 1.15$ is the ST similarity constant for a blast in a uniform medium with the gas adiabatic index $\gamma_g = 5/3$. The ST shock speed can be expressed as

$$U_{ST} = u_0 = \frac{2}{5} \xi_s u_o \left(\frac{t}{t_o} \right)^{-3/5} = \frac{2}{5} \xi_s^{5/2} u_o \left(\frac{M_{ej}}{M_{sw}} \right)^{1/2} = 4.6 \times 10^3 \text{ km s}^{-1} \left(\frac{u_o}{10^4 \text{ km s}^{-1}} \right) \left(\frac{t}{t_o} \right)^{-3/5}. \quad (6)$$

The second to last relation reveals that $U_{ST} \propto \sqrt{E_o/M_{sw}}$, emphasizing that the dynamical state of a ST blast is determined by the blast energy and the mass contained within the blast. Thus, except for numerical factors of order unity, many of the results we present below can be approximately represented independent of the ambient radial density distribution in terms of M_{sw} in combination with E_o and M_{ej} rather than t/t_o and u_o .

Recent X-ray observations of young SNRs reveal in many cases magnetic fields stronger by at least an order of magnitude than the average ISM field (*e.g.*, Bamba et al. 2006; Parizot et al. 2006). The existence of such strong fields is now commonly interpreted as the result of amplification within the shock precursor, either through resonant (Lucek & Bell

2000), or nonresonant (Bell 2004) streaming instability, or possibly hydrodynamic instability driven by the CR pressure gradient in the precursor (Beresnyak, Jones, & Lazarian 2009). The simulations discussed here do not follow self-consistent amplification in the precursor of the magnetic field strength through these processes. Instead, to provide field values in the model SNRs consistent with observations, the upstream magnetic field strengths in SNR models S1 and S2 were set to the relatively large value, $B_0 = 30\mu\text{G}$. For comparison, the S3 and S4 models adopted, $B_0 = 5\mu\text{G}$, which is similar to the mean ISM magnetic field.

In the simulation of the electron synchrotron energy losses and the resulting synchrotron emission, we assume that the field strength is modified during passage through the shock and into the SNR interior in a way that maintains consistency with the diffusion coefficient model (equation [4]). This κ model assumes in the relativistic limit that the scattering length, $\lambda \propto p/(\rho B_0)$, along with a fixed ratio of wave field to total field strength. Consequently, it assumes $B \propto \rho$, so that gas compression through the modified shock, $\chi = \rho_2/\rho_0 \sim 4 - 13$ (see Fig. 2), leads to postshock field values in these models, $B_2 = \chi B_0 \sim 120 - 400 \mu\text{G}$ in S1 and S2 and $\sim 20 - 50 \mu\text{G}$ in S3 and S4.

In all the simulations CR backreaction quickly increases compression from the initial $\chi = 4$ to $\chi \sim 7 - 13$. Subsequently, χ decreases slowly in each case roughly as $\chi \propto t^{-k}$, with $k \sim 1/7 - 1/5$ (see Fig. 2). Kang et al. (2009) found for strong, CR modified, plane shocks that approximately $\chi \propto \mathcal{M}^{1/3}$, where \mathcal{M} is the shock sonic Mach number. For these SNR simulations, $\mathcal{M} \propto t^{-2/5}$, so their result would predict $k \sim 2/15$, which is reasonably consistent. For ST SNRs one has more generally that $\mathcal{M} \propto M_{sw}^{-1/2}$, so for scaling relations discussed in §3 we will assume that $\chi \propto M_{sw}^{-1/6}$.

The SNR simulations start with ST similarity blast waves at a time $t/t_o = 0.5$, because the detailed evolution before that time does not strongly affect the later development. In fact, hydrodynamic simulations of SN blast waves (without the CR terms) show that the evolution of the outer shock speed can be approximated by U_{ST} for $t/t_o > 0.2$, although the true ST solution is established only after the inner reverse shock is reflected at the center at $t/t_o \sim 7$, (*e.g.*, Kang 2006). The simulations are carried out to $t/t_o = 15$, which is deemed sufficient to establish basic ST phase properties of the CR population and their dynamical impact until either the blast strength weakens substantially or the blast becomes radiative. Earlier studies showed that the highest momentum produced by DSA during the expansion of an SNR, p_{max} , is achieved near the end of the free expansion stage, so when $t/t_o \sim 1$, and

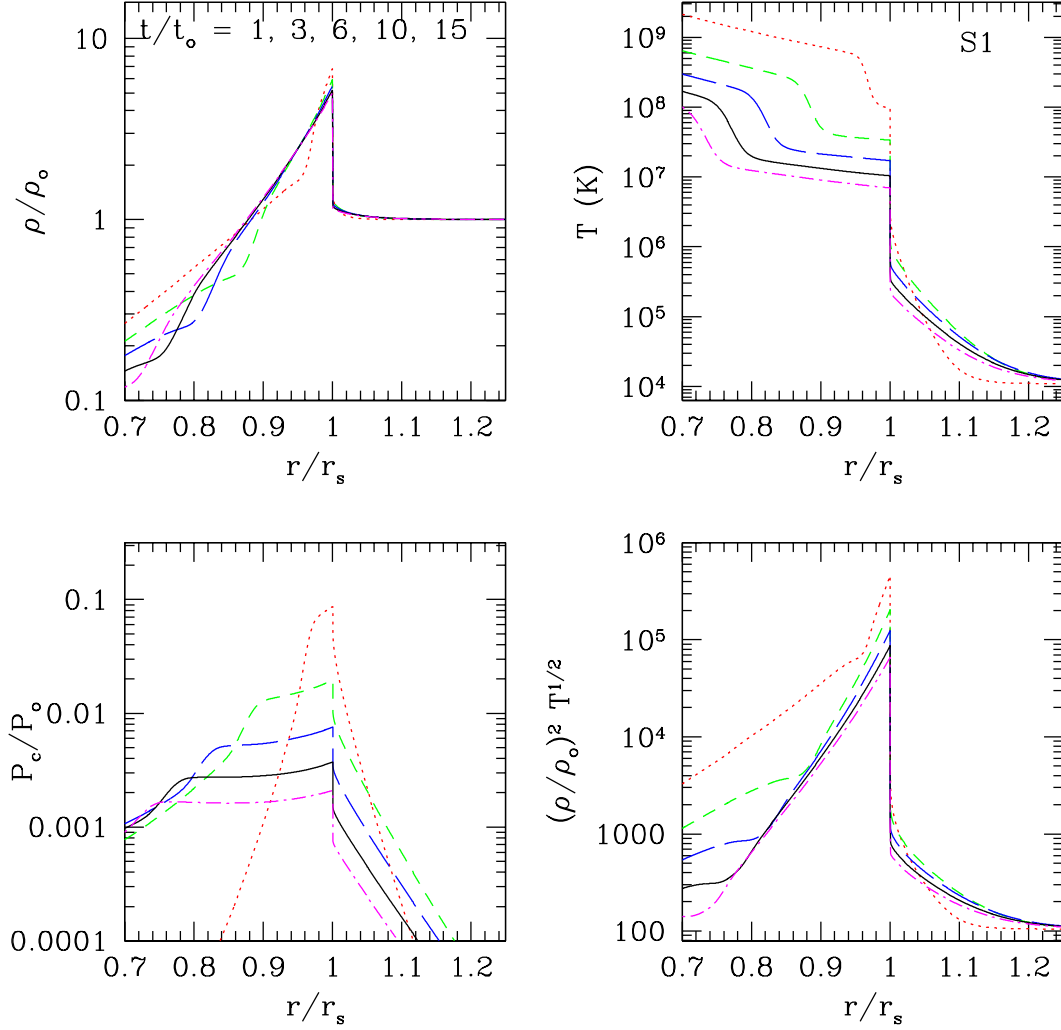


Figure 1. The time evolution of gas density and temperature for model S1 along with CR pressure and the factor $\rho^2 T^{1/2}$, which provides a scaling for thermal bremsstrahlung. Times shown are: $t/t_o = 1$ (red dotted), 3 (green short dashed), 6 (blue long dashed), 10 (black solid), and 15 (magenta dot-dashed). (A colour version of this figure is available in the online journal.)

the transfer of explosion energy to the CR component occurs mostly during the early ST stage, so also when $t/t_o \sim 1$ (*e.g.*, Berezhko & Völk 1997).

2.3 Evolution of the CR modified SNRs

We now summarize the distinctive time evolutionary features of the SNR models used in this study, focusing on nonlinear DSA influences that impact radiative emissions. Fig. 1 shows radial profiles from model S1 of several useful quantities near the shock at times $t/t_o = 1, 3, 6, 10, 15$. The S1 model is representative in this regard of all the simulations. Gas density and temperature are shown in the top two panels. They are combined in the lower right to show $\rho^2 T^{1/2}$ as a proxy for bolometric thermal bremsstrahlung emissivity. The CR

pressure profile is included in the lower left panel. To simplify the plots, initial ST conditions from $t/t_o = 0.5$ are not included. At the first time shown in Fig. 1, $t/t_o = 1$, the shock is modified substantially through compression and heating in the CR precursor. In fact, the CR acceleration and nonlinear modification peak early in the evolution and then decrease in time as the shock slows down (see also Fig. 2).

CR modification of the shock enhanced the postshock density but reduced the gas pressure, leading to a postshock gas temperature lower than that expected for the ST shock near $t/t_o \sim 1$. The effect of CR pressure on the temperature distribution behind the shock can be seen by noting that the slope of temperature rise towards the interior steepens as $P_c(r)$ begins to drop sharply inside the blast. This interior drop in $P_c(r)$ is an artifact of the location of the contact discontinuity separating matter interior and exterior to the initial conditions shock. Its shape depends on the rate of spatial CR diffusion into the blast cavity.

In addition to raising the total density jump within the shock, compression through the precursor also preheats gas before it enters the subshock structure. This adiabatic effect by itself would raise the upstream temperature by a factor $T_1/T_0 \sim (\rho_1/\rho_0)^{2/3}$, or about 1.6 in the S1 model. In fact, Fig. 1 shows that in the S1 model $T_1/T_0 \approx 10 - 100$. The additional heating comes from MHD wave dissipation (equation [3]). Compression through the precursor also decelerates upstream flow before it enters the subshock. All these effects greatly weaken the subshock. So the sonic Mach number of the subshock decreases quickly to $\mathcal{M}_{sub} \approx 4.8$ in the S1 and S4 models and to $\mathcal{M}_{sub} \approx 3.7$ in S3 model by $t/t_o \approx 0.7$ and remains approximately the same afterward, even though the total shock sonic Mach number decreases roughly as $M \propto (t/t_o)^{-3/5}$. Since \mathcal{M}_{sub} stays constant throughout the evolution, the CR injection fraction (ξ) via thermal leakage also stays constant after $t/t_o \approx 0.7$ (see Fig. 2). With the adopted injection parameter ($\epsilon_B = 0.25$), the CR injection fraction is high, $\xi \approx 10^{-3}$, and so the CR acceleration should be close to maximum efficiency.

These shock modifications also significantly influence the thermal bremsstrahlung emission in the SNR, whose distribution is represented by the $\rho^2 T^{1/2}$ profile in the lower right panel of Fig. 1. The thermal bremsstrahlung shell will appear thinner in the strongly modified SNRs than in the analogous gas dynamic SNR. The enhanced postshock density also enhances the thermal bremsstrahlung emissivity. The thinner shell and higher emissivity in the shell combine for a relatively small difference in the bolometric thermal bremsstrahlung luminosity compared to an unmodified shock. The thermal bremsstrahlung luminosity in a fixed spectral band below the spectral cutoff should generally be enhanced, however, since

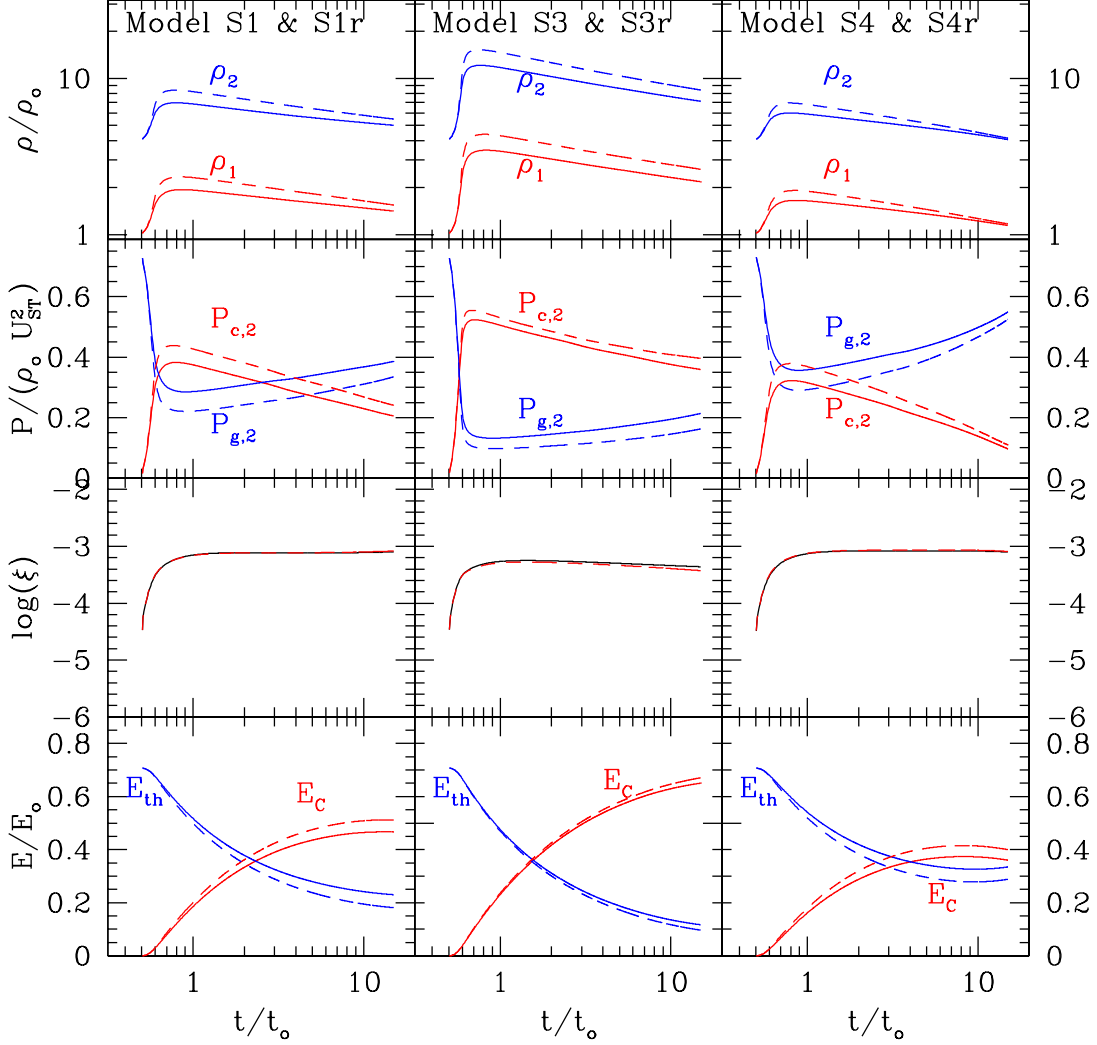


Figure 2. The time evolution of the compression factors, ρ_1/ρ_0 and $\chi = \rho_2/\rho_0$, postshock pressures, $P_{g,2}$ and $P_{c,2}$, the injection fraction, ξ , and the volume integrated energy, E_{th} and E_c is shown for six models. The wave heating parameter is $\omega_H = 0.5$ for S1r, S3r and S4r models, while $\omega_H = 1$ for S1, S3, and S4 models. The r models are the dashed lines. (A colour version of this figure is available in the online journal.)

that has an approximate $1/T$ dependence with respect to the bolometric emission. Of course, the lowered temperatures behind the modified shock also move the upper cutoff of the thermal bremsstrahlung SED from the hard X-ray band into the soft X-ray band.

More insights to the evolutionary behaviors of the S1, S3 and S4 models can be obtained from Fig. 2. As already noted, both the precursor compression, ρ_1/ρ_0 , and the total shock compression, $\chi = \rho_2/\rho_0$, as well as the CR pressure relative to $\rho_0 U_{ST}^2$ increase quickly from their initial values¹, reach their maximum values around $t/t_0 \approx 0.7$, and decrease slowly in time as the shock slows down. In case of the S2 model with higher E_o (not shown in Fig. 2),

¹ The very early evolution is affected by numerical start-up of the simulations.

the CR acceleration and the nonlinear modification are slightly more significant compared to the S1 model, because of a faster shock speed. If the Alfvénic drift and heating were not included, the temperature of the ambient medium (and so the sonic Mach number) would be the primary parameter that determines the CR injection and acceleration efficiencies. Compare, for instance, the S3 model in warm ISM to the S4 model in the hot ISM. Because of the ISM temperature differences, the sonic Mach number of the total shock in the S3 model is 6 times higher than S4 at a given t/t_o . So the S3 SNR would be the more efficient CR accelerator. The low density and high temperature ISM of the S4 model produce low sonic and Alfvénic Mach numbers ($\mathcal{M} \propto T^{-1/2}$, $\mathcal{M}_A = u_s/v_A \propto n_{ISM}^{1/2}/B$), making the CR acceleration much less efficient in that model.

The magnetic field strength also has a significant influence on the SNR evolution even though no direct MHD dynamical effects are modeled. Most important to these simulations, the field strength sets the CR spatial diffusion rate and thus, in concert with u_o , the CR acceleration rate (see §2.4, below). Consequently, particles are accelerated to several times higher momenta (p_{\max}) in the S1 model in comparison to the S3 model because of the enhanced magnetic field and smaller diffusion coefficient (see Fig. 3 below). On the other hand, the stronger field leads to a faster Alfvén speed, which tends to reduce DSA efficiency by adding entropy to the precursor. This reduces the effective velocity jump across the subshock and also reduces the subshock Mach number. Those effects are responsible for the moderate reductions in CR acceleration efficiency in the S1 model compared to the S3 model. The effect of Alfvén wave heating is further illustrated in Fig. 2, where we show three additional simulations, S1r, S3r, and S4r, in which the wave dissipation parameter is reduced from $\omega_H = 1$ to $\omega_H = 0.5$. The smaller wave dissipation rate reduces non-adiabatic heating in the precursor, allowing greater compression through the precursor, so an increase in CR acceleration efficiency.

Fig. 2 also shows the volume integrated thermal energy ($E_{th} = \int 4\pi r^2 dr (3/2) P_g$) and CR energy in units of the SN explosion energy. The energy transfer to CRs seems to saturate for $t/t_o \gtrsim 10$, and the fraction of the blast energy transferred to CRs (E_c/E_o) is about 0.7, 0.5, 0.45, 0.35 in the S3, S2, S1, and S4 models, respectively.

In summary, adopting simple models for Alfvén wave transport, these simulations demonstrate that the CR acceleration would be less efficient at SNRs in hot rarefied ISM where both the sound speed and Alfvén speed are faster, compared to those in a warm phase ISM.

We note that the evolution of CR modified SNRs shown in Fig. 2 (especially, ρ_1 , ρ_2 , $P_{g,2}$

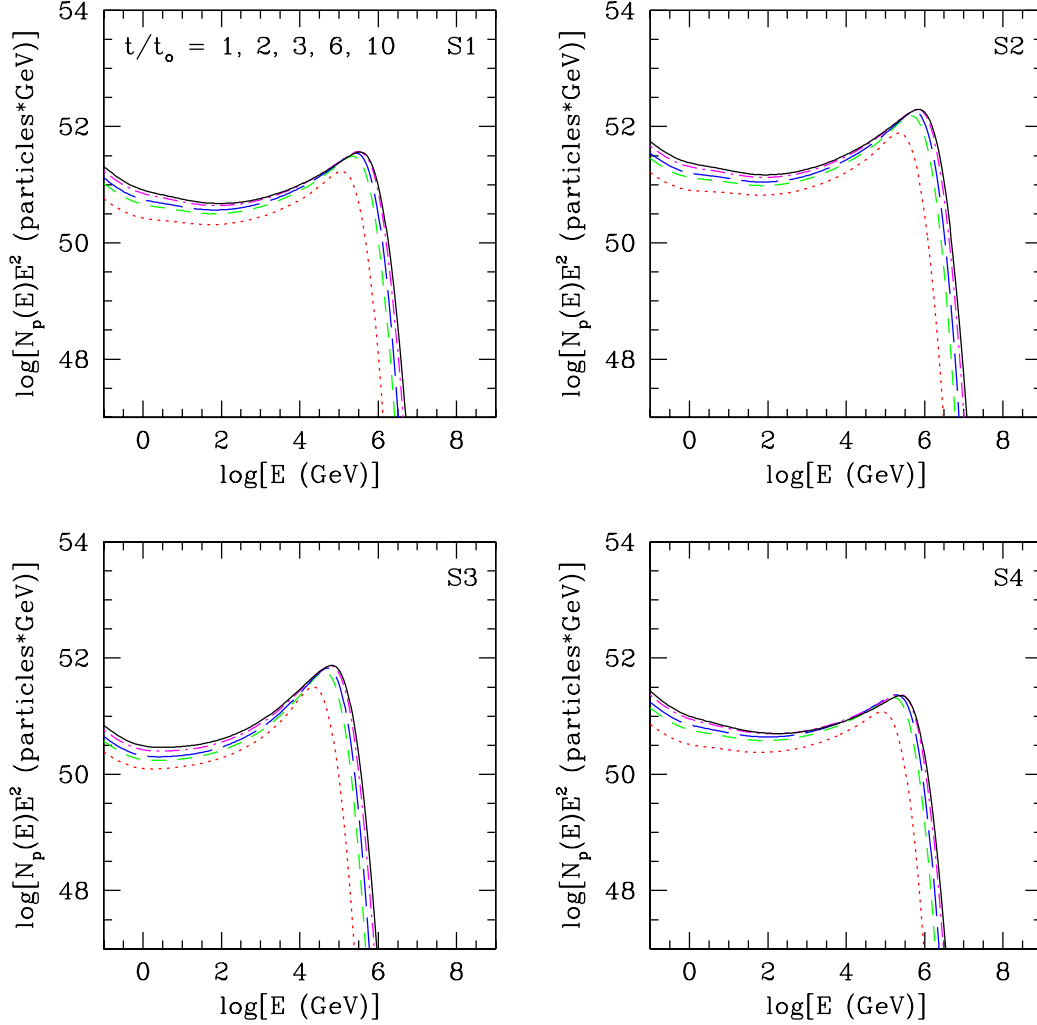


Figure 3. Volume integrated proton spectra in terms of particle kinetic energy at $t/t_0 = 1$ (red dotted), 2 (green short dashed), 3 (blue long dashed), 6 (magenta dot-dashed), and 10 (black solid). (A colour version of this figure is available in the online journal.)

and $P_{c,2}$) differ somewhat from the similar SNR models presented in Kang (2006). Several parameters were different there, *e.g.*, $\mu = 1$, $\epsilon_B = 0.16 - 0.2$, $T_{ISM} = 10^4 \text{K}$ for S1-3 models, and $B_0 = 30 \mu\text{G}$ for the S4 model. The main differences result, however, from a coding error in those earlier simulations, which caused the Alfvénic drift and heating effects to not be fully implemented. The results in the present study are, however, consistent with independent simulations of similar SNRs (Berezhko, Ksenofontov & Völk 2002), in which a Type Ia SNR with $E_o = 3 \times 10^{51} \text{erg}$ and $\xi = 2 \times 10^{-4}$ in warm phase ISM with $n_{ISM} = 0.3 \text{ cm}^{-3}$, $T_{ISM} = 10^4 \text{K}$, and $B_0 = 20 \mu\text{G}$ was calculated.

2.4 The CR proton and electron populations

Fig. 3 shows the volume integrated CR proton energy spectrum, $E^2 \cdot N_p(E, t) = \int 4\pi r^2 dr E^2 n_p(E, r, t)$, where $E = m_p c^2 (\sqrt{1 + p^2} - 1)$, is the proton kinetic energy, and $n_p(E, r, t) dE = 4\pi p^2 f(p) dp = n(p, r, t) dp$. We note that in the models with enhanced B_0 (*i.e.*, S1 and S2) particles of charge Z can be accelerated to $E_{\max} \approx Z \cdot 10^{15} \text{eV}$ by the early ST stage. This is consistent with results of other analogous SNR simulations (*e.g.*, Berezhko & Völk 2007). In the early ST stage the shock structure is significantly modified, with $\chi = \rho_2/\rho_0 \sim 7 - 10$, so $E^2 \cdot N(E)$ is concave upwards. This well-known behavior is a consequence of momentum dependent diffusion across the precursor; CRs near the injection momentum experience only the sub-shock velocity jump, while higher momentum CRs see a larger velocity jump. Thus, the CR spectrum is softer at lower momentum and harder just below the high energy cutoff than the strong shock test particle result, $N(E) \propto E^{-2}$. Moreover, the Alfvénic drift in the precursor further softens the spectrum of newly accelerated CRs as the shock slows down and the Alfvénic Mach number decreases.

It will be useful below to estimate the maximum CR proton momentum expected at a given time t/t_o . This we do by integrating the standard expression for the average momentum gained per unit time for a particle (Lagage & Cesarsky 1983),

$$\begin{aligned} \frac{dp}{dt} &= \frac{u_0(1 - \mathcal{M}_A^{-1}) - u_2}{3} \left(\frac{\kappa_0}{u_0(1 - \mathcal{M}_A^{-1})} + \frac{\kappa_2}{u_2} \right)^{-1} p, \\ &= \frac{[u_0(1 - \mathcal{M}_A^{-1})]^2}{q_0 \kappa_0} \frac{p}{(1 + \frac{\kappa_2}{\kappa_0} \frac{q_0}{q_0 - 3})}, \end{aligned} \quad (7)$$

where, once again, subscripts 0 and 2 refer to upstream and downstream conditions in the shock frame, respectively. The useful parameter,

$$q_0 = 3 \frac{u_0 - v_A}{u_0 - v_A - u_2} = 3 \frac{\chi(1 - \mathcal{M}_A^{-1})}{\chi(1 - \mathcal{M}_A^{-1}) - 1} \approx \frac{3\chi}{\chi - 1}, \quad (8)$$

is the DSA test particle spectral index, including the upstream Alfvén wave drift term, $|u_w| = v_A = u_0/\mathcal{M}_A$, for consistency with equation (1). The right-most expression for q_0 neglects this term, since it is a relatively minor correction in our model SNRs.

Neglecting the Alfvén wave term and assuming a constant compression ratio, χ , but including the diminishing shock speed in the ST solution, equation (8) gives for protons accelerated between t_i and t ,

$$p_{\max} \approx \frac{0.54 u_o^2 t_o}{\kappa_n} \frac{1}{q_0} \left(\frac{t_o}{t_i} \right)^{0.2} \left[1 - \left(\frac{t_i}{t} \right)^{0.2} \right]. \quad (9)$$

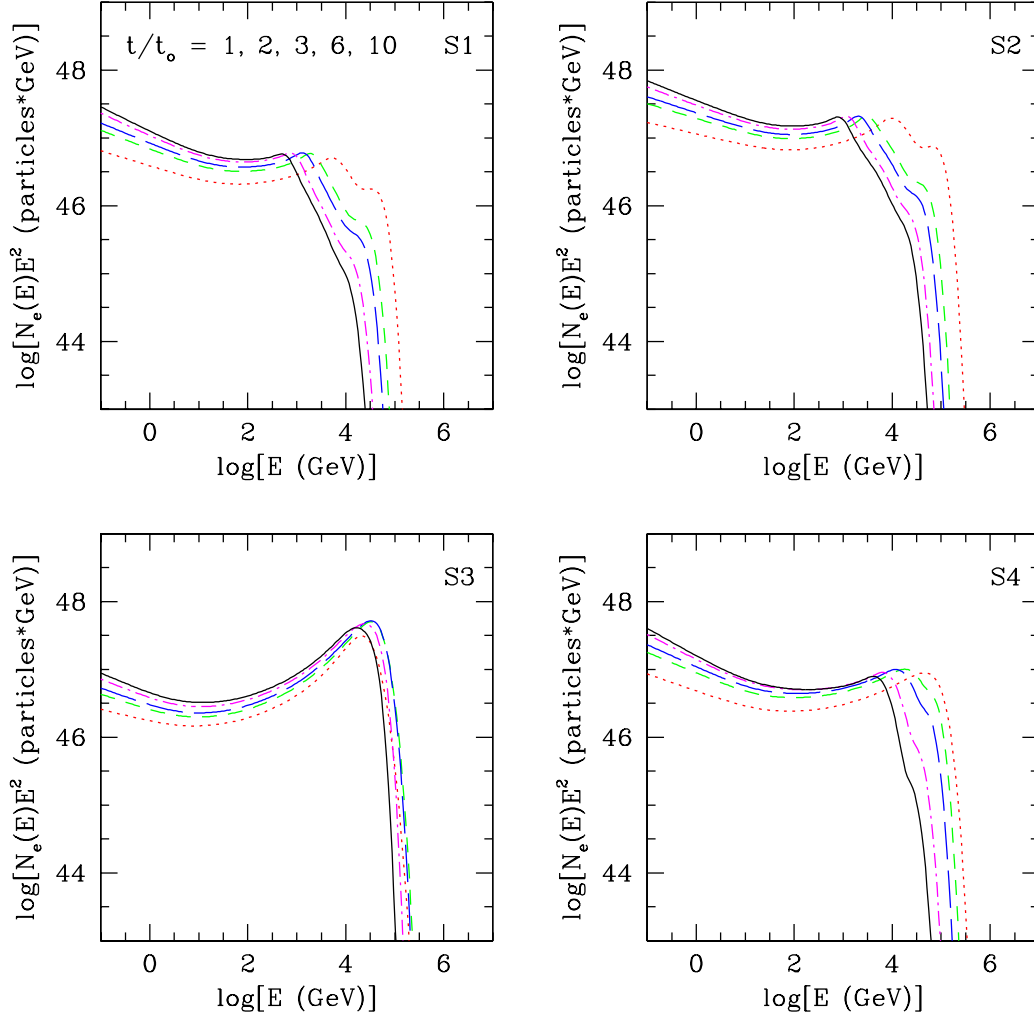


Figure 4. Volume integrated electron spectra in terms of particle kinetic energy at $t/t_o = 1$ (red dotted), 2 (green short dashed), 3 (blue long dashed), 6 (magenta dot-dashed), and 10 (black solid). (A colour version of this figure is available in the online journal.)

For our simulations, which start from $t_i/t_o = 0.5$, equation (9) asymptotes to $p_{\max} \approx 0.15(t_o u_o^2/\kappa_n) \approx 3.8 \times 10^{13}(E_o/10^{51})^{1/2}B_\mu n_{ISM}^{-1/3}$ eV/c at large t/t_o using $\chi = 4$ for convenience. At $t/t_o = 10$ this gives, for example, $p_{\max} \approx 7.8 \times 10^{14}$ eV/c for the S1 model and $p_{\max} \approx 5.8 \times 10^{14}$ eV/c for the S4 model. These estimates are consistent with the numerically determined cutoff energies in $E^2 \cdot N_p(E)$ shown in Fig. 3.

The CR electron spectra develop substantial differences from the proton spectra, despite their common DSA interactions (*e.g.*, Webb, et al. 1984; Berezhko, Ksenofontov & Völk 2009; Blasi 2010; Zirakashvili & Aharonian 2010). Fig. 4 shows the volume integrated CR electron energy spectra, $E^2 \cdot N_e(E, t) = 4\pi \int r^2 E^2 \cdot n_e(E, r, t) dr$, at several times for each SNR model. Note that the electron spectra cutoff at much lower energies than the analogous

proton spectra shown in Fig. 3. The structures of the integrated electron spectra are complex around and below the cutoff. The low energy break in each spectrum identifies the energy at which the synchrotron/Compton loss time for electrons downstream of the shock equals the SNR age. Below this energy radiative energy losses are negligible. The integrated spectrum between this break and the high energy cutoff is controlled by the downward movement of the electron energy cutoff as recently accelerated electrons move into the SNR interior away from the shock. The high energy cutoff itself develops at the shock and corresponds to the energy where synchrotron losses balance DSA gains.

At the shock the electron cutoff momentum, p_{ec} , can be evaluated by balancing DSA and radiative losses. In particular, p_{ec} corresponds to the momentum at which the average momentum gain by an electron in one pair of shock crossings, $\Delta p_{DSA} = (4/3)p[u_0(1 - \mathcal{M}_A^{-1}) - u_2]/v$, equals the momentum loss from radiation during the same period of time, $\Delta p_r = b_2 4\kappa_2/[u_2 v] + b_0 4\kappa_0/[(u_0(1 - \mathcal{M}_A^{-1})v)]$, where $v = c\sqrt{1 - (m_e/(m_p p))^2}$ is the electron velocity, with $b_0(p)$ and $b_2(p)$ defined by equation (3) in terms of the upstream and downstream effective magnetic fields, respectively (*e.g.*, Webb, et al. 1984). This leads to the (test particle) result,

$$p_{ec} = \left(\frac{m_e}{m_p}\right)^{1/2} \left(\frac{6\pi m_e c^2}{\sigma_T c B_{eff,0}^2}\right)^{1/2} \frac{(1 - \mathcal{M}_A^{-1})u_s}{\sqrt{\kappa_n q_0 \left[1 + \frac{1}{\chi} \frac{B_{eff,2}^2}{B_{eff,0}^2} \frac{q_0}{q_0 - 3}\right]}}, \quad (10)$$

$$\approx \left(\frac{m_e}{m_p}\right)^{1/2} \left(\frac{6\pi m_e c^2}{\sigma_T c B_0^2}\right)^{1/2} \frac{(1 - \mathcal{M}_A^{-1})u_s}{\sqrt{\kappa_n q_0 \left[1 + \chi \frac{q_0}{q_0 - 3}\right]}},$$

where $u_s(t) = u_0$ is the instantaneous shock speed. Here we assume $\kappa_0/\kappa_2 = B_2/B_0 = \chi$ and the second expression corresponds to the limit, $B_{eff} \approx B$, which is applicable in our simulations. This p_{ec} translates into a cutoff energy for CR electrons at the shock,

$$E_{ec} \approx 3.3 \text{ PeV} \frac{1}{\sqrt{B_{0,\mu} q_0 (1 + \chi \frac{q_0}{q_0 - 3})}} \left(\frac{u_s(t)}{10^4 \text{ km s}^{-1}}\right), \quad (11)$$

where the subscript μ on B indicates field strength in microGauss.

Equation (11) evaluated with $B_{0,\mu} = 30$ and $\chi = 4$ predicts $E_{ec} \sim 100$ TeV for these models at time $t \sim t_o$, which is consistent with the simulation results shown in Fig. 4. This is also consistent with the upper limits on electron spectra derived from observation for a number of observed SNRs (Reynolds & Keohane 1999) and with SNR simulation results, for example, of Berezhko, Ksenofontov & Völk (2002). During the early ST stage, the maximum acceleration momentum gradually saturates as given in equation (9), but then it eventually

asymptotes to p_{ec} determined by the instantaneous shock speed as given in equation (11). As a result, for larger values of B_0 , the electron cutoff energy is smaller, but the maximum proton energy is larger.

Electrons advected into the SNR interior will continue to lose radiative energy with the combined synchrotron/IC radiative cooling time, $t_r(p) = p/b(p)$, given by

$$t_r(p) = 133 \text{ yrs} \left(\frac{B_{\text{eff}}}{100 \mu\text{G}} \right)^{-2} \left(\frac{p_{ec}}{10^4} \right)^{-1}. \quad (12)$$

Setting the cooling time behind the shock to the SNR age, $t_r(p_{e,1}) = t$, provides a conservative, rough estimate for the minimum electron momentum influenced by radiative cooling,

$$p_{eb} \sim 1.3 \times 10^3 \left(\frac{B_{\text{eff}}}{100 \mu\text{G}} \right)^{-2} \left(\frac{10^3 \text{ yrs}}{t} \right). \quad (13)$$

Recall that $E_e \approx p \text{ GeV}$. Below p_{eb} the integrated electron spectrum should be very similar in form to the proton spectrum at the same momentum, consistent with results shown in Fig. 3 and Fig. 4.

To help interpret the integrated electron spectra, $N_e(E)$, between the break momentum p_{eb} and the cutoff momentum p_{ec} , we note in the thin shell approximation that $N_e(E) \propto n_e(E)r^2\Delta r$, and that the local electron spectrum, $n_e(p) = K_{e/p}n_p(p)$, below p_{eb} . In this case it is easy to show that the integrated electron energy spectrum below E_{ec} would be steeper by about one in the spectral index than the integrated proton spectrum in the same energy range. The steepened slope would extend down to the energy $E_{eb} \approx p_{eb} \text{ GeV}$. On the other hand, in spherical flows following roughly the ST behavior, the postshock flow accelerates away from the high magnetic field region just inside the shock while reducing the magnetic field strength. These effects reduce the rate of cooling from that implied in equation 13, thus shortening the interval between p_{ec} and p_{eb} . Then the spectral steepening of the integrated electron population compared to the proton spectrum will be greater than unity, as observed in our results.

3 MODELING OF CONTINUUM EMISSIONS FROM THE SNR MODELS

3.1 The Cosmicp code for radiative processes

The nonthermal radio to γ -ray emissions expected from CR electrons and from proton secondary products in the simulated SNRs were computed through the post processing of model data using *Cosmicp*, a general purpose nonthermal continuum emissions code developed in-house around published radiative process and inelastic particle scattering formulations.

Cosmicp computes direct emissions from input electron/positron populations and calculates the inelastic collision products for photon and proton interactions with matter, including pion and lepton secondaries. Lepton secondaries are incorporated into *Cosmicp*, but they do not make significant contributions to SNR emissions, so are not included in our analysis here. *Cosmicp* was designed to be very flexible, so makes as few assumptions as possible. For example, the input energy distributions of energetic particles are arbitrary. Using inputs $n(E, r)$ for CR electrons and protons along with the relevant environmental information, such as magnetic field, radiation field, and gas density, *Cosmicp* calculates the volume emissivity per unit frequency, $j_\nu(\nu, r)$, for each radiative process, where ν is the emitted photon frequency. Bremsstrahlung from the thermal electron population is also included in our calculations with *Cosmicp*, but electronic emissions involving discrete atomic transitions are excluded. All calculations are in cgs units, which allows easy linking to actual observables.

Specific CR electronic emissions incorporated into our analysis here include synchrotron, inverse Compton (IC), and bremsstrahlung processes. The synchrotron, IC, and relativistic bremsstrahlung formalisms followed presentations in Schlickeiser (2002). The nonrelativistic bremsstrahlung formalism used to compute the spectrum due to the thermal population followed Jackson (1999). The IC formalism uses an arbitrary, user supplied spectral form for the ambient photon field, so *Cosmicp* can model interactions with multiple blackbody or non-blackbody sources. It does, however, currently assume an isotropic incident radiation field.

Hadronic interactions include, as noted, inelastic proton-proton and photon-proton interactions. Photopion production in the latter case can be calculated from an arbitrary photon field using the formalism in Kelner & Aharonian (2008). Pion generation from proton-proton interactions uses relations presented in Kelner, Aharonian, & Bugayov (2006). The principal radiative process that results from hadronic interactions in our SNR context is secondary π^0 decay, which is dominated by the 2γ channel. *Cosmicp* can include helium in the hadron-hadron production of photons. However, that correction was not applied in the present calculations. For a CR composition similar to that incident on the earth and an ISM with normal metallicity this correction would increase the pion production rate by $\sim 25\%$ (Schlickeiser 2002).

3.2 Nonthermal emissions: Local interaction conditions

Rates for nonthermal electron bremsstrahlung and p-p collisions leading to pion decay γ 's depend on the product of the local plasma density and the CR electron and proton density, respectively. These are taken in our emission calculations directly from the simulations. Synchrotron and IC emissions depend on the spatial magnetic field distribution and the ambient radiation field respectively. As discussed in §2.2, we assume the local magnetic field strength scales with thermal plasma density; i.e., $B(r, t) = B_0[\rho(r, t)/\rho_0]$. The ambient photon field for our calculations is uniform and models the Galactic radiation field in the solar neighborhood as given by Schlickeiser (2002). It consists of a sum of four components: the Cosmic Microwave Background (CMB) blackbody at 2.7 K with an energy density of 0.25 eV/cm^3 , dust at 20 K with an energy density of 0.4 eV/cm^3 , old yellow stars at 5000 K with an energy density of 0.3 eV/cm^3 , and young blue stars at 20,000 K with an energy density of 0.09 eV/cm^3 . The total energy density for the ambient photon field is 1.04 eV/cm^3 . In evaluating equation (12) for electron energy losses this translates to an effective magnetic field of $B_r = 6.5 \mu\text{G}$. We point out in §3.3.2, however, that IC scattering by the highest energy electrons of all but the CMB and dust emission radiation fields is limited by electron recoil, and reduced by the Klein-Nishina cutoff, so this value overestimates the effective B_r . In our models electron losses are dominated by the synchrotron process, so this is a relatively small correction, in any case.

3.3 Volume integrated radiation spectrum from the model remnants

A general impression of the radiative emissions produced in the simulated SN Ia remnants can be obtained from Fig. 5, which shows for the S1 simulation at $t/t_0 = 3$ the volume integrated Spectral Energy Distribution (SED), νL_ν from radio frequencies to PeV γ -rays. Synchrotron emission dominates the spectrum for $\nu \lesssim 10^{20} \text{ Hz}^2$. There is a narrow γ -ray window, $10^{20} \text{ Hz} \lesssim \nu \lesssim 10^{21} \text{ Hz}$ dominated by bremsstrahlung from CR electrons interacting with thermal plasma. Above $\sim 10^{21} \text{ Hz}$ ($E_\gamma \approx 400 \text{ keV}$) the spectrum is dominated by photons from π^0 decays for the ISM density assumed in the simulation.

The spatially integrated radiative emissions at selected times for all four models are shown in Fig. 6. Independent of time, the radiation is mostly dominated by the electronic CR emissions at lower energies, but, for the most part, proton π^0 decay at high energies.

² Useful translation factors are: $1 \text{ keV} \rightarrow 2.42 \times 10^{17} \text{ Hz}$, $10^{17} \text{ Hz} \rightarrow 0.41 \text{ keV}$.

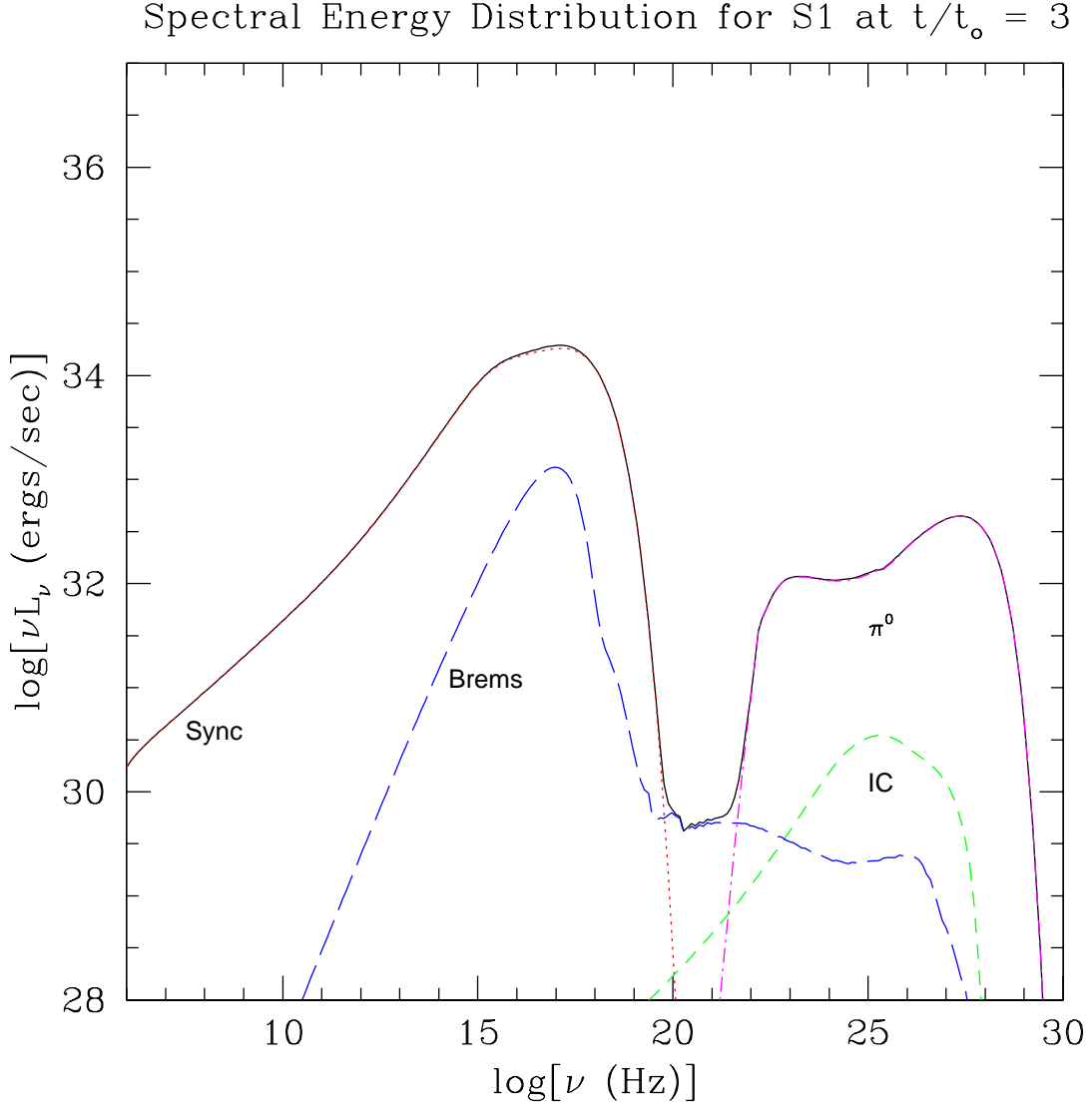


Figure 5. The volume integrated SED for the S1 model at $t/t_o = 3$: total emission (solid black line), synchrotron emission (red dotted line), IC emission (green short dashed line) bremsstrahlung (blue long dashed line), γ s from π^0 decays (magenta dot-dashed line). (A colour version of this figure is available in the online journal.)

The additional bump in the UV and X-ray seen at late times especially in model S1 is due to thermal bremsstrahlung. This feature appears as the high energy end of the electron distribution is depleted by radiative losses, revealing thermal emission from hot, postshock plasma. The lower gas density in the S4 model reduces this thermal contribution, and also the nonthermal bremsstrahlung γ -rays.

In the S4 model the γ -ray portion of the spectrum is a composite, dominated by π^0 decays between $\nu \sim 10^{22} - 10^{23}$ Hz ($E_\gamma \sim 40$ MeV - 400 MeV) and again above about $\nu \sim 10^{28}$ Hz ($E_\gamma \sim 40$ TeV). However, the lower ambient gas density of the S4 model also reduces the π^0 production enough that IC emissions are dominant between the synchrotron

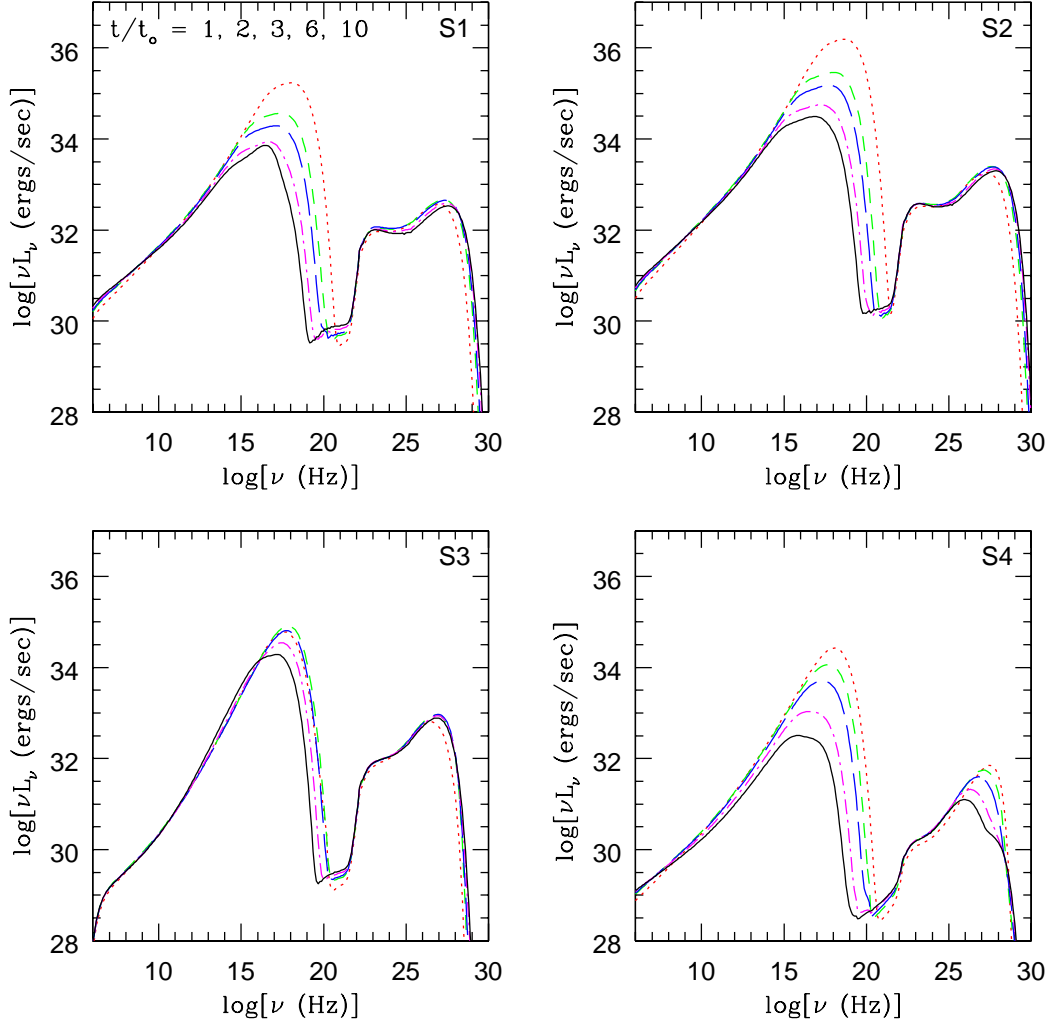


Figure 6. The total volume integrated SEDs for models S1-S4 over time. The dotted red line is $t/t_o = 1$, the short dashed green line is 2, the long dashed blue line is 3, the dot-dashed magenta line is 6, and the solid black line is 10. (A colour version of this figure is available in the online journal.)

cutoff and $\sim 10^{22}$ Hz. IC emission is also predominant between $\nu \sim 10^{24} - 10^{27}$ Hz ($E_\gamma \sim 4$ GeV - 4 TeV).

We outline below some useful relations for interpretation of the calculated radiative emissions in the context of these models, including some simple scaling relations that can be helpful in extending our results to different model parameter choices than we have used. We begin with discussions of the electronic emissions and follow with the essential elements to understand the calculated emissions produced by inelastic p-p collisions.

3.3.1 Electron Synchrotron Emission

The electron synchrotron emission spans a frequency range from radio to roughly the emission peak for electrons near the Lorentz factor $\gamma_{ec} \approx (m_p/m_e)p_{ec}$. Above this energy the electron population is heavily modified by losses and begins to drop off severely. The associated synchrotron peak frequency is $\nu_{sc} \approx 0.3 \nu_{crit}$, where

$$\nu_{crit} = \frac{3}{2} \nu_B \gamma_{ec}^2 \sin(\theta) = 4.2 \times 10^{16} \text{ Hz} \left(\frac{B}{100 \mu\text{G}} \right) \left(\frac{\gamma_{ec}}{10^7} \right)^2 \sin(\theta), \quad (14)$$

and ν_B is the nonrelativistic electron cyclotron frequency (Blumenthal & Gould 1970). For the calculations presented here we set the angle of the magnetic field with respect to the observer, $\theta = 90^\circ$.

Using equations (11) for the upper electron energy cutoff and (14) we can derive the following estimate for the cutoff frequency in the synchrotron spectrum in the limit that $B_{eff} \approx B$; namely,

$$\nu_{sc} \approx 1.8 \times 10^{20} \text{ Hz} \frac{1}{q_0(1 + \chi \frac{q_0}{q_0-3})} \left(\frac{u_s(t)}{10^4 \text{ km s}^{-1}} \right)^2. \quad (15)$$

For $\chi = 4$ and $q_0 = 4$ the factor $q_0[1 + \chi q_0/(q_0 - 3)] = 17$, so $\nu_{sc} \approx 3 \times 10^{18} \text{ Hz} (u_s/10^4 \text{ km/s})^2$, consistent with the results in Fig. 6. As noted by previous authors (*e.g.*, Bamba et al. 2003; Berezhko & Völk 2004) the cutoff frequency is determined mainly the instantaneous shock speed and independent of magnetic field strength except through its implicit dependence on the factor χ . This explains the similarity between the values for the synchrotron cutoffs for S1 and S3 even though they have very different magnetic fields.

The break frequency that corresponds to the estimated break momentum p_{eb} depends only on magnetic field strength and the shock age as

$$\nu_{sb} \approx 7.52 \times 10^{14} \text{ Hz} \left(\frac{10^3 \text{ yrs}}{t} \right)^2 \left(\frac{B_{eff}}{100 \mu\text{G}} \right)^{-3} \quad (16)$$

The values of ν_{sc} and ν_{sb} generated from equation (15) and (16), respectively, are consistent with the SEDs shown in Figs. 5 and 6. The photon energies corresponding to the synchrotron cutoff frequency fall in the range 0.1 – 10 keV, consistent with observed Type Ia remnants (Reynolds & Keohane 1999).

The synchrotron luminosities of all four simulations below the X-ray cutoffs can be approximately related to each other by a simple scaling relation that, in addition, provides a means to extend these results approximately to different parameter choices. For

$E < 10^2$ GeV the volume integrated electron energy spectrum shown in Fig. 4 can each be represented by

$$N_e(E, t) \approx (s - 1) \mathbf{N}_e E_i^{s-1} E^{-s}, \quad (17)$$

where $E_i \sim k_B T_2$ represents the suprathermal injection energy of CR electrons, \mathbf{N}_e is the volume integrated number of CR electrons and s is the energy distribution mean power law slope between E_i and $< E < 10^2$ GeV. For reference, recall that momentum and energy distribution power law indices are related as, $s = q - 2$. The synchrotron luminosity can then be expressed approximately as (Blumenthal & Gould 1970)

$$\nu L_{\nu, \text{sync}} \propto \mathbf{N}_e E_i^{s-1} B^{(s+1)/2} \nu^{-(s-3)/2}. \quad (18)$$

Since $\mathbf{N}_e \propto \xi K_{e/p} M_{sw}$ and the injection fraction ξ is nearly a constant for $t/t_o \gtrsim 1$ in these simulations, $\mathbf{N}_e \propto M_{sw} \propto (t/t_o)^{6/5}$. The injection energy, E_i scales roughly with the shock speed, so also as $E_i \propto M_{sw}^{-1/2}$. In §2.2, we established the approximate scaling, $\chi \propto M_{sw}^{-1/6}$, so $B \propto M_{sw}^{-1/6}$. Then $\nu L_{\nu, \text{sync}} \propto M_{sw}^{(17-7s)/12}$ in these simulations. For example, $\nu L_{\nu, \text{sync}} \propto M_{sw}^{1/4}$ for $s = 2$ or $\nu L_{\nu, \text{sync}} \propto M_{sw}^{1/18}$ for $s = 7/3$. Since the index s varies slowly within the range $9/5 \lesssim s \lesssim 7/3$ in these simulations, the radio luminosity should depend very weakly on M_{sw} or t/t_o , consistent with the results in Fig. 6.

Note that equation (18) alone is not sufficient to estimate synchrotron emission behaviors within the X-ray band. The synchrotron emission in this band is generated predominantly by electrons near the spectral break ν_{sb} and cutoff ν_{sc} . Consequently, X-ray synchrotron emissions are heavily influenced by the positions of these frequencies.

3.3.2 Electron Bremsstrahlung and IC Emission

The bremsstrahlung and IC contributions are mostly subdominant in our models. At early times synchrotron emission dominates thermal bremsstrahlung over the bands where they both contribute. The bremsstrahlung peak is roughly $\nu_T \sim kT_2/h$, while the synchrotron cutoff, ν_{sc} , is given by equation (15). Both cutoffs depend primarily on the shock speed squared, so evolve more or less together. The decreasing role of synchrotron emission in the X-ray band results, on the other hand, from the fact that CR electron energy losses downstream of the shock cause that population to become further depleted, dropping the relative emitted power between ν_{sc} and ν_{sb} , revealing the thermal bremsstrahlung emission.

Consequently for $t/t_o \gtrsim 6$ in S1 thermal bremsstrahlung is an important X-ray contributor (see Fig. 6).

In another exception to the general emission behaviors, we note that IC emissions near $h\nu \sim 1$ TeV in model S4 exceed the otherwise dominant pion decay gamma-rays. The S4 π_0 decay emissions, on the other hand, are smaller by two orders of magnitude, reflecting the similarly lower S4 ambient density (see equation [24]).

Some comments on the contributions and form of the IC SED may be useful. As noted in §3.2 the IC emissions result from a combination of incident photon fields with blackbody forms at different characteristic temperatures. The effective incident field is just their sum. While the CMB is a blackbody in both spectral form and energy density, the radiation from dust and stars has greatly diluted energy density compared to the Planck functions for their respective temperatures. In each case we can assign a frequency-independent dilution factor, D , given by the ratio of the local energy density to the appropriate Planck function. The dilution factors for the radiation field properties listed in §3.2 are: for the CMB, $D_{CMB} = 1$, for thermal dust emission, $D_d = 5.3 \times 10^{-4}$, yellow stars, $D_{y*} = 1.0 \times 10^{-13}$, and for blue stars, $D_{b*} = 1.2 \times 10^{-16}$.

The IC scattering source function in the limit of Thomson scattering of incident photons with a Planck spectrum with color temperature, T_i , and dilution factor, D_i , is

$$q_{IC,i}(E_\gamma \gg k_B T_i \gamma_{ec}^2) = \frac{r_0^2 (s-1) p_i^{s-1}}{\pi \hbar^3 c^2} K_{e/p} \mathbf{n}_p F(s) \left(\frac{m_p}{m_e} \right)^{s-1} (k_B T_i)^{(s+5)/2} D_i E_\gamma^{-(s+1)/2} \quad (19)$$

where r_0 is the classical electron radius, $F(s)$ is a dimensionless scaling function as defined in Blumenthal & Gould (1970); for $s \sim 2$ $F(s) \sim 5$. We have used the form of the electron spectrum given in equation (17).

For a representative CR electron slope, $s = 2$, the different background radiation components contribute in equation (19) according to factors $\propto T_i^{7/2} D_i$, which leads to a ranked list $CMB, d, y*, b*$ represented in the ratios $1 : 0.58 : 0.03 : 0.004$. This suggests that the CMB and the dust IR radiation are the predominant contributors to the IC SED, despite the roughly comparable photon energy densities of the stellar components (see §3.2). In this regard it is also worth noticing that the IC spectral peak at a few TeV involves photons scattered to energies close to E_{ec} , the maximum possible, whereas the Thomson scattering limit $E_\gamma \sim k_B T_i \gamma_{ec}^2$ would take photons into the PeV range when scattered from the incident stellar photon fields. In fact, photons from those higher temperature incident fields are scattered in the Klein-Nishina limit by the most energetic electrons, as given by the condition

on the incident photon energy, $h\nu > m_e c^2 / \gamma_{ec}$. This condition can be expressed practically as

$$T_i > \frac{(m_e c^2)^2}{k_B E_{ec}} \approx 3.0 \times 10^3 \left(\frac{\text{TeV}}{E_{ec}} \right) \text{ Kelvin.} \quad (20)$$

For $E_{ec} \sim 100$ TeV only the CMB and, marginally, the dust radiation escape this limit. The scattering cross section in the Klein-Nishina limit is reduced from the Thomson cross section, so that, in fact, gamma ray IC emissions in these models are dominated by the CMB and dust emissions.

We can use equation (19) to construct an approximate, simple scaling relation for the IC luminosity, $\nu L_{\nu, IC}$, in the Thomson regime assuming an electron distribution of the form in equation (17),

$$\nu L_{\nu, IC} \propto \mathbf{N}_e E_i^{s-1} u_{rad}^{(s+5)/8} \nu^{-(s-3)/2} \quad (21)$$

where the radiation energy density, u_{rad} , would be approximated by the sum of the CMB and dust radiation densities. The IC luminosity, $\nu L_{\nu, IC}$, scales as $M_{sw}^{(3-s)/2}$, similar to the synchrotron luminosity. In fact, the ratio of the two for emissions at synchrotron frequency $\nu_{sync} \ll \gamma_{ec}^2 \nu_B$, and IC frequency, $\nu_{IC} \ll \gamma_{ec}^2 \nu_{Td}$ in, say, the radio and gamma ray bands, is just

$$\frac{\nu L_{\nu, IC}}{\nu L_{\nu, sync}} \propto \frac{u_{rad}^{(s+5)/2}}{u_B^{(s+1)/4}} \left(\frac{\nu_{sync}}{\nu_{IC}} \right)^{(s-3)/2} \quad (22)$$

This indicates that the ratio between the two contributions to the SED for each of our SNR models remains roughly constant over time. Since u_{rad} is model independent, the ratio of IC gamma ray luminosity to radio synchrotron luminosity varies in these models roughly as $\propto 1/B_o^{3/2}$.

3.3.3 π^0 Decay Gamma-Ray Radiation

Gamma ray decay products from inelastic p-p collisions, and $\pi_0 \rightarrow 2\gamma$, in particular, are commonly viewed as the “smoking gun” for hadronic DSA in SNRs. As noted above these emissions mostly dominate the gamma ray SEDs in the SNR models being discussed here. Although the detailed modeling of π^0 decay gamma rays is fairly complicated, a simple analytic approximation offered by Pfrommer & Enßlin (2003) provides a good understanding of the π^0 gamma ray SED in these models. In particular, if the local CR proton energy spectrum above the threshold for pion production ($E > 780\text{MeV}$) is a power law, $n_p(E) = (s-1)\mathbf{n}_p E_i^{s-1} E^{-s}$, then the γ -ray source function, q_π , (photons/time/volume/energy) is

$$q_\pi \approx \frac{\sigma_{pp} c}{m_p c^2} 2^{2-s} \frac{4}{3s} \left(\frac{m_\pi}{m_p} \right)^{-s} \left[\left(\frac{2E_\gamma}{m_\pi c^2} \right)^\delta + \left(\frac{2E_\gamma}{m_\pi c^2} \right)^{-\delta} \right]^{-s/\delta} n (s-1) E_i^{s-1} \mathbf{n}_p \quad (23)$$

where $\sigma_{pp} \approx 3.2 \times (0.96 + e^{4.4-2.4s}) \times 10^{-26} \text{ cm}^2$ is an effective p-p cross-section for pion production, $\delta = 0.14s^{-1.6} + 0.44$, while n is the density of the thermal protons, and $m_\pi = 135 \text{ MeV}/c^2$ is the π^0 mass.

This source function peaks for $E_\gamma = m_\pi c^2/2 = 67.5 \text{ MeV}$. At low energies it asymptotes to a power law $q_\pi \propto E_\gamma^s$, while it asymptotes at high energies to a power-law $q_\pi \propto E_\gamma^{-s}$. The source function cuts off above roughly 10% the proton high energy cutoff (Kelner et al. 2006). The radiated power per unit volume (the SED) will scale with $E_\gamma^2 q_\pi$. Using equation (23) for $s > 2$ the SED would peak around $E_\gamma = 67.5 \text{ MeV}$ $[(\delta + 2\delta/s)/(\delta - 2\delta/s)]^{1/(2\delta)}$. There is no SED maximum for $s \leq 2$, except at the high energy cutoff.

Assuming a proton spectrum of the form in equation (17) one obtains a simple high energy scaling relation for the volume integrated pion decay luminosity

$$\nu L_{\nu, \pi^0} \propto n \mathbf{N}_p E_i^{s-1} E_\gamma^{2-s} \propto \xi M_{sw} E_i^{s-1} n E_\gamma^{2-s}. \quad (24)$$

This scaling is consistent with the results shown in Fig. 6, showing in particular why the pion decay emissions are similar in all the models except for S4 because of its lower n_{ISM} .

The proton distributions in these SNR models are not really true power laws, of course, but have concave upwards forms; that is, s decreases with energy. Protons near the pion production thresholds just above a GeV are mostly responsible for gamma rays near the low energy gamma ray peak. In that range typically $s \sim 2.3$ (see Figs. 3 and 4). The expected π^0 gamma ray SED peak is near 1 GeV ($\nu \sim 2 \times 10^{23} \text{ Hz}$), consistent with Figs. 5 and 6. The model π^0 SEDs in Figs. 5 and 6 also exhibit a second, higher energy peak. That results from the concave upwards character of the proton spectra and their eventual cutoffs around or above a PeV for $t/t_o > 1$. Except for model S4 the proton spectra have $s < 2$ below their cutoffs, making their upper π^0 gamma ray peaks quite prominent.

Since the the upper π^0 gamma ray peak is so prominent it can be used to determine the peak proton energy in the SNR (*e.g.*, Aharonian et al. 2007). From the gamma-ray production relations in Kelner et al. (2006) one can see that the peak in gamma-ray production for a monoenergetic proton of $E_p > 1 \text{ TeV}$ occurs at a photon energy $E_\gamma \approx 0.1 E_p$. Looking at Fig. 6 for S1-S3 the gamma-ray peak energy is around about 40 – 400 TeV ($10^{28} - 10^{29} \text{ Hz}$) which corresponds to a proton energy of 400 – 4000 TeV. This is consistent with the peaks in the proton spectra in Fig. 3.

Both IC and π^0 source functions depend on the number of CRs accelerated, while the pion-generated gamma rays also depend on the density of the thermal plasma. The ratio of q_π/q_{IC} found from equations (23) and (19),

$$\frac{q_\pi}{q_{IC}} = \pi \frac{\sigma_{pp}}{r_0^2} \frac{\hbar^3 c^3}{(k_B T)^2 m_e c^2} \frac{4 \cdot 2^{2-s}}{3s} \left(\frac{m_e}{m_\pi} \right)^s \left(\frac{E_\gamma}{k_B T} \right)^{(s+1)/2} \cdot \left[\left(\frac{2E_\gamma}{m_\pi c^2} \right)^\delta + \left(\frac{2E_\gamma}{m_\pi c^2} \right)^{-\delta} \right]^{-s/\delta} \frac{n}{DF(s)K_{e/p}}, \quad (25)$$

provides a useful comparison of the two processes for estimation and scaling purposes.

Again, taking $s = 2$ as representative, then including CMB and dust contributions in the radiation field as discussed earlier, and setting $K_{e/p} = 10^{-4}$ along with $n = 2 \text{ cm}^{-3}$, appropriate to postshock conditions for model S1 (see Table 1 and Fig. 2), we can use equation (25) to estimate

$$\frac{q_\pi}{q_{IC}} \sim 400 \left(\frac{E_\gamma}{400 \text{ MeV}} \right)^{-1/2} \left(\frac{n}{2 \text{ cm}^{-3}} \right) \left(\frac{10^{-4}}{K_{e/p}} \right), \quad (26)$$

which is valid roughly for $E_\gamma \sim 400 \text{ MeV}$ ($\nu \sim 10^{23} \text{ Hz}$). This number is in reasonable agreement with the numerical results displayed in Fig. 5. Equation (26) suggests that π^0 decay emissions would need to be reduced by at least two orders of magnitude from those in model S1 before IC emissions would become dominant in the GeV band. Note for scaling purposes from equation (25) that $q_\pi/q_{IC} \propto n/K_{e,p}$. Consequently, GeV IC emissions would dominate when $n_{ISM} \lesssim 30 K_{e,p} \text{ cm}^{-3}$, consistent with the model S4 results as displayed in Fig. 6.

3.4 X-ray and gamma-ray brightness distributions

Since X-ray telescopes and the new generations of gamma-ray telescopes have the ability to resolve images of SNRs with distances inside some tens of kpc, it is valuable to model spatial brightness profiles of these emissions. The profiles also provide insights to the similarities and differences among the different emission processes. The spatial profiles for all 4 models look very similar, so we will focus on model S1. The top panels of Fig. 7 show at several times the model S1 radial brightness profiles of broadband fluxes in the *Chandra* X-ray band (2 - 10 keV) (Garmire et al. 2003) and the High Energy Stereoscopic System (HESS) gamma-ray band (140 GeV - 40 TeV) (Berge 2008). The profiles are plotted against the radius perpendicular to the line of sight from the center of the SNR, what we will call the projected radius R , relative to the projected shock radius, $R_s(t)$. The lower panels of Fig.

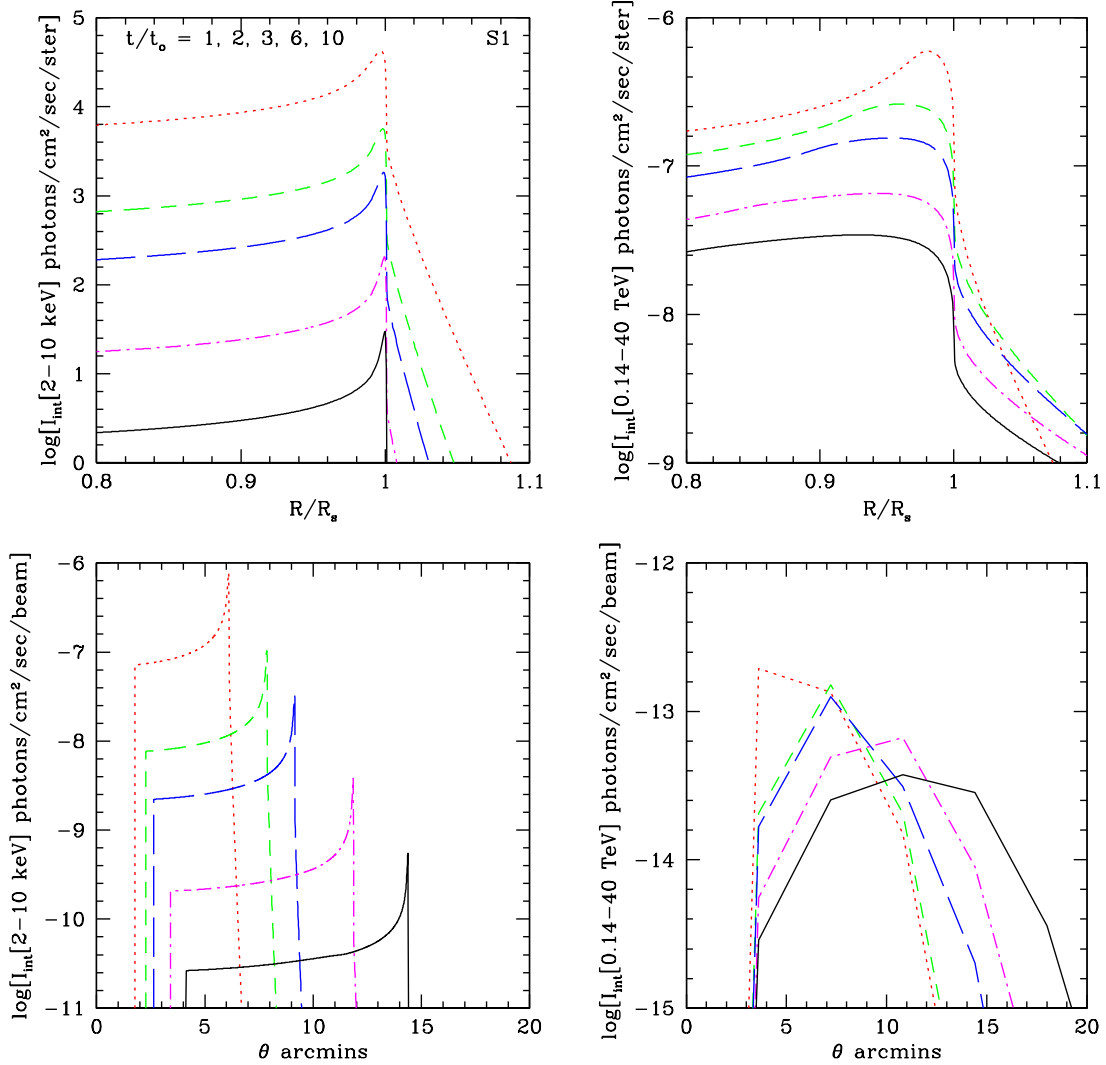


Figure 7. The radial brightness profiles for the S1 model in the *Chandra* X-ray band (2 - 10 keV) and HESS band (140 GeV - 40 TeV). Upper panels: Brightness profiles in terms of projected radius, R/R_s . Lower panels: The same profiles in angular units convolved to the resolution of *Chandra* and HESS with the SNR at 2 kpc. The line types are the same as Fig. 5. (A colour version of this figure is available in the online journal.)

7 present the same profiles in angular units when the SNR is placed at a distance of 2 kpc and the image has been convolved to the nominal resolutions in these bands for *Chandra* and HESS.

Since the exponential cutoff of the synchrotron radiation spectrum, $E_\gamma(\nu_{sc}) \propto (t/t_o)^{-6/5}$ lies in the *Chandra* band, the 2 - 10 keV X-ray fluxes decrease quickly in time. This is evident in the evolution of the X-ray brightness profiles in Fig. 7.

The X-ray brightness profiles are also very narrowly peaked at the projected shock position. The X-ray profiles in Fig. 7 have characteristic apparent FWHM (Full Width Half

Max) of $\sim 1\%$. This is consistent with X-ray observations of several young SNRs that have revealed thin nonthermal filaments at the forward shock with a FWHM thickness $\sim 1\%$ of the shock radius in projection (Bamba et al. 2003; Ballet 2006; Parizot et al. 2006). One can estimate the observed thickness of the shock rim when radiative losses are the dominant factor by the following relation from Parizot et al. (2006), $\Delta R_{obs} = 4.6u_2t_r$, where t_r is defined in equation (12), and the factor 4.6 is a geometrical correction that comes from projecting a spherical shock with an exponential emission profile onto a plane and taking the FWHM of the resulting profile (Ballet 2006). Taking $u_2 = U_{ST}/\chi$, the ratio of shock speed to shock radius for the ST similarity solution, $U_{ST}/r_{ST} = (2/5)/t$, the relative FWHM shock thickness can be approximated as

$$\frac{\Delta R_{obs}}{R_s} \approx \frac{1.84}{\chi} \left(\frac{t_r}{t} \right) = \frac{0.11}{\chi} \left(\frac{B_{eff}}{200\mu\text{G}} \right)^{-2} \left(\frac{\gamma_{ec}}{2 \times 10^7} \right)^{-1} \left(\frac{t}{500\text{yrs}} \right)^{-1}, \quad (27)$$

where the numerical evaluation of t_r from equation (12) has been applied on the RHS. Taking $\chi \sim 7$ for young SNR along with characteristic values for the rest of parameters, equation (27) indeed predicts $\Delta R_{obs}/R_s \sim 1\%$. We note, in addition, applying equation 11 in equation 27 that $\Delta R_{obs} \propto B^{-3/2}$, independent of time or the SNR shock speed ((see, also, e.g., Völk, Berezhko, & Ksenofontov 2005)).

The gamma-ray spatial profile in Fig. 7 is dominated by π^0 decay and is very broadly peaked at the shock. The peak at the shock is due to the emissivity scaling as the product of the CR density and the thermal gas density. Roughly, this will resemble $j_{\nu,\pi} \propto \rho^2$, once again similar to thermal bremsstrahlung. However, the brightness profile is considerably broader than the emissivity owing to same projection effects noted earlier for the X-ray profile.

The IC spatial profile, which is mostly buried under the neutral pion radiation in these models, will have a similar profile to the X-ray synchrotron, because they both come from the same population of electrons. Thus the same argument for why the synchrotron rim should be thin would apply to the IC spatial profile. However, the X-ray synchrotron profile will be slightly thinner than the IC profile owing to its dependence on the magnetic field. Since the IC rim will be thin and the π^0 decay rim will be thick one could in theory differentiate between the two scenarios by looking at the shock thickness. If the shock is thinner in the TeV gamma-rays than the thermal gas distribution, like the X-ray synchrotron profile, then IC is dominant. But if the γ -ray profile has a thickness similar to the gas profile thickness, then π^0 decays are dominant. This level of discrimination, unfortunately, will not be available in the near future.

The lower panels of Fig. 7 provide a look at the S1 model SNR placed at a distance of 2 kpc, about the distance to SN 1006. The intensity profiles have been convolved, as outlined below, to the angular resolution of the *Chandra* ACIS (Advanced CCD Imaging Spectrometer) in the X-ray (0.5 arcsec Garmire et al. (2003)), and HESS around one TeV (3.6 arcmin Berge (2008)). The *Chandra* ACIS has a diffuse flux sensitivity (3σ detection) of 1.6×10^{-5} counts/cm²/sec/arcsec² or about 1.3×10^{-5} counts/cm²/sec/beam for 10 ksec of observing time. Our model X-ray rims would be fully resolved by Chandra at the 2 kpc distance, although none of the earliest profile shown would be detected with a 10 ksec ACIS exposure.

HESS resolution is sufficient to identify a shell-like structure of the gamma-ray profile, but would not be able to resolve the width of the rim, since only a few beams would fit across the source. The HESS diffuse flux sensitivity (8σ detection) of about 2.3×10^{-14} counts/cm²/sec/arcmin² or about 9.3×10^{-13} counts/cm²/sec/beam for 103 hours (Garmire et al. 2003; Berezhko, Ksenofontov & Völkel 2009) would be sufficient to detect this model source at each time shown. Detailed comparison to models of the gamma-ray rim will require the higher resolution of next generation of Imaging Atmospheric Cherenkov Telescope's (IACT) such as the Cerenkov Telescope Array (CTA) (Wagner et al. 2009).

3.5 Comparisons to real SNRs

We round out this discussion by referencing the emissions from these model SNRs to some real Type Ia SNRs. In doing so we emphasize that the models S1-S4 were not intended to reproduce any particular SNR, but rather to represent properties of CR modified Type Ia SNRs evolving in typical environments during the ST stage. The following exercise aims not to establish precise physical properties of the selected real SNRs, but to illustrate how broadband observations, along with simple scaling relations, can be used in conjunction with such models for a qualitative assessment of the physical state of an observed remnant. The potential for such applications is growing greatly with each new generation of high energy observatory. For instance several SNRs have been observed by IACTs at TeV photon energies, now including the Type Ia SN 1006 (Acero et al. 2010) and Tycho (Acciari et al. 2011). Similarly, *Fermi* is collecting an impressive catalog of SNRs detected in the 0.2 – 300 GeV gamma ray band (*e.g.*, Abdo et al. 2010). *Fermi* has not reported detection of any Type Ia SNRs so far. However, the core collapse SNR, Cas A, which has been seen at a distance of

3.4 kpc, provides a useful benchmark. *Fermi* measured a flux, $\nu F_\nu \approx 5 \times 10^{-12}$ ergs/cm²/sec at 12.2σ significance after 396 days of all sky observation (Abdo et al. 2010). This translates to an isotropic luminosity, $\nu L_\nu = 7 \times 10^{33}$ ergs/s. Over an equivalent time *Fermi* would not detect our remnants at the distance of Cas A. However *Fermi* is scheduled to observe in all sky mode for at least 5 years, so it could marginally detect ($\sim 3\sigma$) models S1-S3 at $t/t_o \sim 10$ at the distance of Cas A.

As noted in §3.3.3, the TeV gamma-ray spectra of these model SNRs extend to photon energies well above the upper limits for IACT's such as HESS of ~ 40 TeV. The proposed High Altitude Water Cherenkov experiment (HAWC) would be able to reach above this limit and would have a 5σ detection threshold for photons above 1 TeV of $\nu F_\nu \approx 1 \times 10^{-12}$ ergs/cm²/sec for 1 year (González 2008). At a distance of 3 kpc that translates to a luminosity of $\nu L_\nu = 10^{33}$ ergs/s. This would allow HAWC to detect all of our models excluding S4 at that distance after 1 year.

For the other three primary bands, radio, X-ray and TeV γ -ray, actual observations of Type Ia SNRs already exist. We use the scaling relations in the previous sections to compare our models to two well observed Type Ia SNRs; SN 1006 and Tycho. We caution again that the simulations were idealized, so we should not expect close matches. Still, the comparisons may offer simple insights into the dynamical states of the SNRs and their CR populations.

3.5.1 SN 1006

SN 1006 lies at a distance of about 2.2 kpc and at that distance had a radius of 9.6 pc as of 1993 ($t=987$ yrs) (Berezhko, Ksenofontov & Völk 2009). Ambient density estimates vary, but the recent proper-motion-based dynamical estimate, $n_{ISM} \approx 0.085$ cm⁻³, Katsuda et al. (2009) is representative. In order to establish a dynamical state for the comparison we utilize the ST expressions in §2.2 to obtain

$$\xi_s = \left(\frac{r_s}{r_o}\right) \left(\frac{t_o}{t}\right)^{2/5} = \frac{r_s}{t^{2/5}} \left(\frac{\rho_o}{E_o}\right)^{1/5}. \quad (28)$$

The age, t , is well constrained, and the shock radius, r_s , is relatively well constrained by observation. Thus, even though ρ_o and E_o are not individually as well constrained, their ratio ρ_o/E_o is fixed at least within the ST paradigm. Since the explosion energy is likely close to 10^{51} erg (one “FOE”), by assuming an ejected mass, $M_{ej} = 1.4M_\odot$, we can express the ambient density constraint as $n_{ISM} \approx 0.037 E_{FOE} \text{ cm}^{-3}$, consistent with the Katsuda et al. (2009) estimate. Similarly, $t_o = 520 E_{FOE}^{-5/6}$ yrs, so at the observed time, $t/t_o = 1.9 E_{FOE}^{5/6}$.

The swept up mass, $M_{sw} \approx 4.6M_{\odot}E_{FOE}$. We can use our simulation S1 for comparison with SN 1006.

The model S1 1.4 GHz radio luminosity, $\nu L_{\nu} \approx 3 \times 10^{31}E_{FOE}$ ergs/sec. The measured 1.4 GHz flux of SN1006 is $\nu F_{\nu} \approx 2.2 \times 10^{-13}$ ergs/cm²/sec (Reynolds & Ellison 1992), which translates to an isotropic luminosity, $\nu L_{\nu} \approx 1.3 \times 10^{32}$ ergs/sec, so slightly larger than the model. As noted in §3.3.1 the model radio emission is almost independent of time, or swept up mass, but it does scale with the electron injection efficiency, $K_{e/p}$ and the magnetic field, B , as $K_{e/p}B^{3/2}$ (assuming $s = 2$). So, a modest increase in this combination would bring the two luminosities into satisfactory agreement.

The expected X-ray synchrotron luminosity is set within the model by the radio luminosity and by the break frequency, $\nu_{sb} \propto E_o/M_{sw}^{5/3}$. For model S1 at $t/t_o \approx 2$ the break frequency, $\nu_{sb} \sim 10^{17}$ Hz, compared to the “roll-off” frequency in SN 1006 estimated by Bamba et al. (2008) to be $\nu_{roll} \sim 6 \times 10^{16}$ Hz. Fig. 6 shows the model S1 X-ray luminosity near the break to be roughly 10^{34} erg/sec. The measured 0.1 – 2 keV (mostly nonthermal) X-ray flux of SN 1006 is $\nu F_{\nu} \approx 1.42 \times 10^{-10}$ ergs/cm²/sec (Allen, Petre & Gotthelf 2001). This translates to an isotropic luminosity, $\nu L_{\nu} \approx 8.1 \times 10^{34}$ ergs/sec. The X-ray comparison is consistent with the radio comparison; the best match would come from a modest increase in $K_{e/p}B^{3/2}$.

HESS has reported detection of VHE gamma-rays from SN 1006 with a flux above 1 TeV gamma ray flux, $\nu F_{\nu} \approx 3 \times 10^{-13}$ ergs/cm²/sec (Acero et al. 2010). This translates to a luminosity of $\nu L_{\nu} \approx 2 \times 10^{32}$ ergs/sec. This is very close to the (pion-decay) luminosity for model S1 shown in Fig. 6.

3.5.2 *Tycho*

Tycho’s SNR may not yet have reached the ST dynamical stage (Völk, Berezhko, & Ksenofontov 2008), but we include it in order to illustrate another application of the scaling relations. In this context we note that Kang (2006) found evolution of the outer, blast wave, shock to be represented reasonably well by ST scaling well before the actual ST phase begins.

Tycho has a distance that is variously estimated, but mostly less than about 3.6 kpc. We will parameterize our comparison including a distance factor $d_{3.6} = d/3.6kpc$. The corresponding radius in 1983 (t=411 yrs) was $r_s = 4 d_{3.6}$ pc (Völk, Berezhko, & Ksenofontov 2008). The SNR lies in the plane of the galaxy in a warm ISM region of estimated density ~ 0.3 cm⁻³. The warm ISM context makes models S1 - S3 the most appropriate compar-

ison. Since we have established the validity of the emission scaling relations among these models, we focus on model S1. We follow the same procedure as for SN 1006, except we now include distance as an adjustable parameter in order to illustrate its influence on the comparison. Equation (28) leads in this case to $n_{ISM} = 0.43 E_{FOE}/d_{3.6}^5 \text{ cm}^{-3}$ along with $t/t_o = 1.8 E_{FOE}^{5/6}/d_{3.6}^{5/3}$ and $M_{sw} \approx 4.3 M_{\odot} E_{FOE}/d_{3.6}^2$.

Tycho's measured 1.4 GHz radio flux is $\nu F_{\nu} \approx 1.6 \times 10^{-13} \text{ ergs/cm}^2/\text{sec}$ (Reynolds & Ellison 1992), which translates to an isotropic luminosity, $\nu L_{\nu} \approx 2.5 \times 10^{32} d_{3.6}^2 \text{ ergs/sec}$. The scaling given in equation (18) using nominal parameters leads to a model S1 1.4 GHz luminosity estimate appropriate for Tycho, $\nu L_{\nu} \approx 4.9 \times 10^{31} E_{FOE}/d_{3.6}^2 \text{ ergs/sec}$, or about five times smaller than the observed luminosity. If we wanted to achieve a better match using the full scaling from equation (18), we would increase the factor $\xi K_{e,p} B^{3/2} E_{FOE}/d_{3.6}^4$ by this factor of five.

Additional information comes from the X-ray and γ -ray portions of the SED. Tycho's measured 10 keV X-ray flux is $\nu F_{\nu} \approx 3.2 \times 10^{-11} \text{ ergs/cm}^2/\text{sec}$ (Allen, Gotthelf & Petre 1999). This translates to an isotropic luminosity, $\nu L_{\nu} \approx 4.9 \times 10^{34} d_{3.6}^2 \text{ ergs/sec}$. The model S1 flux at 10 keV would be for nominal parameters, $\nu L_{\nu/sync} \approx 10^{34} E_{FOE}/d_{3.6}^2 \text{ erg/sec}$. This is once again about a factor five smaller, so leads to a consistent way to compare the model to the observations. The Very Energetic Radiation Imaging Telescope Array System (VERITAS) group recently reported a γ -ray flux above 1 TeV Acciari et al. (2011), $\nu F_{\nu} \approx 3 \times 10^{-13} \text{ ergs/cm}^2/\text{sec}$. This translates to an isotropic luminosity, $\nu L_{\nu} \approx 4.7 \times 10^{32} d_{3.6}^2 \text{ erg/sec}$, consistent with the properties of our model S1.

4 SUMMARY

Using kinetic, nonlinear DSA simulations, we have calculated the energy spectrum of CR protons and electrons accelerated at blast waves from Type Ia supernovae and the emission contributions of those cosmic rays during Sedov-Taylor phase. The DSA model is based on the assumptions that streaming instability amplified Alfvén waves lead to Bohm-like diffusion, and that particle escape at the highest energy is negligible in the time frame of interest. With the CR proton injection rate from the thermal plasma, $\xi \sim 10^{-3}$, obtained here, the strong SNR shocks become significantly modified by the formation of a CR precursor that pre-compresses the inflowing plasma. This both weakens the dissipative subshock and, combined with Bohm diffusion, leads to a CR proton spectrum that is steeper at low energies

than the strong shock test particle form, $N_p(E)dE \propto E^{-2}dE$, but flatter at high energies. In addition, the drift of scattering centers with respect to the bulk plasma also softens the CR spectrum and the dissipation of scattering waves reduces the CR acceleration efficiency. These effects become more important for stronger upstream magnetic fields and for a lower ISM density, because the Alfvén speed is $v_A \propto B/\sqrt{n_{ISM}}$.

We assume the electron to proton ratio is $K_{e/p} = 10^{-4}$ that is, $n_e(p_{inj}) = K_{e/p}n_p(p_{inj})$ near the injection momentum. Although the value of $K_{e/p} \sim 10^{-2}$ is implied by the Galactic CR flux observed at Earth, the smaller value is preferred for source spectra that best fit multi-band photon spectra of several young SNRs (*e.g.*, Berezhko, Ksenofontov & Völk 2009; Morlino, Amato & Blasi 2009; Zirakashvili & Aharonian 2010). However, since the electrons are passive with respect to the hydrodynamics the results of this paper can be rescaled to any (small) $K_{e/p}$.

The thermal and nonthermal continuum emissions from the model SNRs were computed using a radiative process code *Cosmicp*. The spectra are dominated in most cases by CR electron emissions in the X-ray band and below. Gamma-ray emissions were dominated in most cases by $p-p$ produced π^0 decay. For the hot, low density ISM case inverse Compton electronic emissions dominated the TeV γ -ray band, however. We note of course that the final comparison of hadronic versus leptonic origin of GeV TeV γ -rays depends on the ambient matter density and the electron to proton ratio as $n_{ISM}/K_{e/p}$ among other parameters.

We presented simple scaling relations for the synchrotron, IC and π^0 decay luminosities that seem to provide appropriate approximate means to rescale the numerical results to different times and model parameters. We applied these relations as simple tests to SN 1006 and Tycho’s SNR demonstrating a basic consistency and a means to explore parameter space dependencies.

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REFERENCES

- Abdo A., Ackermann M., Ajello M. et al., 2010, *ApJ*, 710, L92
- Acciari, V. A., Aliu, E., Arlen, T. et al., 2011, *ApJ*, 730, L20
- Acero, F., et al., 2010, *A&A*, 516, 62
- Aharonian F. A., Akhperjanian A. G., Bazer-Bachi A. R. et al., 2007, *A&A*, 464, 235
- Allen G. E., Gotthelf E. V., Petre R., 1999, *Proc. 26th ICRC(Salt Lake City)*, 3, 480
- Allen G. E., Petre R., Gotthelf E. V., 2001, *ApJ*, 558, 739
- Ballet, J. 2006, *Advances in Space Research*, 37, 1902
- Bamba A., Yamazaki R., Ueno M., Koyama K., 2003, *ApJ*, 589, 827
- Bamba A., Yamazaki R., Yoshida T., Terazawa T., Koyama K., 2006, *Adv. Space Res.*, 37, 1439
- Bamba A., et al., 2008, *PASJ*, 60, S153
- Bell A.R., 2004, *MNRAS*, 353, 550
- Beresnyak A., Jones T. W., Lazarian A., 2009, *ApJ*. 707, 1541
- Berezhko E.G., Yelshin V.K., & Ksenofontov L.T. 1994, *A.Ph.*, 2, 215
- Berezhko E. G., Völk H. J., 1997, *A.Ph.*, 7, 183
- Berezhko E. G., Ksenofontov L.T., Völk, H. J., 2002, *A&A*, 395, 943
- Berezhko E. G., Völk H. J., 2004, *A&A*, 427, 525
- Berezhko E. G., & Völk, H. J., 2007, *ApJ*, 661, 175
- Berezhko E. G., Ksenofontov L.T., Völk, H. J., 2009, *A&A*, 505, 169
- Berge D., et al., 2008, *Proc. 30th ICRC(Merida)*, 2, 679
- Blandford R. D., Ostriker J. P., 1978, *ApJ*, 221, L29
- Blandford R. D., Eichler D., 1987, *Phys. Rep.*, 154, 1
- Blasi. P., 2010, *MNRAS*, 402, 2807
- Blumenthal G. R., Gould R. J., 1970, *RMP*, 42, 237
- Caprioli, D., Blasi, P., & Amato, E. 2011, *Astroparticle Physics*, 34, 447
- Drury L. O’C., Ellison D. E., Aharonian F. A. et al., 2001, *Space Science Reviews*, 99, 329
- Ellison D. C., Cassam-Chenai, G., 2005, *ApJ*, 632, 920

- Ellison D. C., Patnaude D. J., Slane P., Blasi P., Gabici S., 2007, *ApJ*, 661, 879
- Funk S., Hinton J. A., Moriguchi Y. et al., 2007, *A&A*, 470, 249
- Gaisser T. K., 2005, *Neutrinos and Explosive Events in the Universe*, Springer, p. 3
- Garmire G. P., Bautz M. W., Ford P. G., Nousek J. A., Ricker G. R. Jr, 2003, *Proc. SPIE*, 4851, 28
- Gieseler U. D. J., Jones T. W., Kang H., 2000, *A&A*, 364, 911
- González M. M., 2008, *Proc. 30th ICRC(Merida)*, 3, 1563
- Hillas A. M., 2005, *Journal of Physics G*, 31, R95
- Hillas A. M., 2006, *arXiv:astro-ph/0607109*
- Jackson J. D., 1999, *Classical Electrodynamics*, 3rd Ed., John Wiley & Sons Inc.
- Jones T. W., 1993, *ApJ*, 619, 619
- Kang H., Jones T. W., Gieseler U. D. J., 2002, *ApJ*, 579, 337
- Kang H., Jones, T. W. 2006, *A.Ph.*, 25, 246
- Kang H., 2006, *Journal of Korean Astronomical Society*, 39, 95
- Kang, H., Ryu, D. & Jones, T. W., 2009, *ApJ*, 695, 1273
- Katsuda, S., Petre, R., Long, K. S., Reynolds, S. P., Winkler, P. F., Mori, K. & Tsunemi, H., 2009, *Ap.J.*, 692, L105
- Kelner S. R., Aharonian F. A., 2008, *Phys. Rev. D*, 78, 034013
- Kelner S. R., Aharonian F. A., Bugayov, V. V., 2006, *Phys. Rev. D*, 74, 034018
- Lagage P. O., Cesarsky C. J., 1983, *A&A*, 118, 223
- Lemoine-Goumard M., Aharonian F. A., Berge D., Degrange B., Hauser D., Komin N., Reimer O., Schwanke U., 2007, *Ap&SS*, 309, 379L
- Lucek S. G., Bell A. R., 2000, *MNRAS*, 314, 65
- Markiewicz W. J., Drury L. O'C., Völk H. J., 1992, *A&A*, 236, 487
- Malkov M.A., Völk H. J., 1998, *Adv. Space Res.*, 21, 551
- Malkov M.A., Drury, L.O'C., 2001, *Rep. Progr. Phys.*, 64, 429
- McKenzie J. F., Völk H. J., 1982, *A&A*, 116, 191
- Morlino G., Amato E., Blasi P., 2009, *MNRAS*, 392, 240
- Parizot E., Marcowith A., Ballet J., Gallant Y. A., 2006, *A&A*, 453, 387
- Pfrommer C., Enßlin T. A., 2003, *A&A*, 407, L73
- Ptuskin, V. S., Zirakashvili, V. N., Seo, E.S. 2010, *ApJ* 718, 31
- Reynolds S. P., Ellison D. C., 1992, *ApJ*, 399, L75
- Reynolds S. P., Keohane, J. W., 1999, *ApJ*, 525, 368

- Reynolds S. P., 2008, ARA&A, 46, 89
- Schlickeiser R., 2002, Cosmic Ray Astrophysics, Springer
- Skilling J., 1975, MNRAS, 172, 557
- Uchiyama Y., Aharonian F. A., Tanaka T., Takahashi T., Maeda, Y., 2007, Nature, 449, 576
- Völk H. J., Berezhko E. G., Ksenofontov L. T., 2005, A&A, 433, 229
- Völk H. J., Berezhko E. G., Ksenofontov L. T., 2008, A&A, 483, 529
- Wagner R. M., Lindfors E., Sillanpää A., Wagner S., 2009, Proc. 2009 Fermi Symposium, eConf C0911022. Stanford, California
- Webb, G. M., Drury, L. O'C. & Biermann, P., 1984, A&A, 137, 185
- Zirakashvili V. N., Aharonian F. A., 2010, ApJ, 708, 965